EDDY-CURRENT LOSSES IN THE PERMANENT MAGNETS OF A PM MACHINE

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ABSTRACT

A gas turbine driven, high-speed, high-efficiency generator system intended for use in series-hybrid vehicles is developed. It consists of a permanent-magnet (PM) generator and a rectifier.
The eddy-current loss in the magnets of the high-speed PM generator may be problematic. Therefore, a model of the PM machine including this loss in the magnets due to the time harmonics of the stator currents is introduced and verified. This loss can be represented by magnet loss resistances in the equivalent circuits. It can be decreased by using smaller magnets or by using sinusoidal stator currents.

INTRODUCTION

Objective of the research project. The aim of the research project is the development of a gas turbine driven high-speed, high-efficiency generator system. This generator system is intended for use in series-hybrid vehicles, the drive system of which is depicted in figure 1. It can also be used in aircraft, in vessels, in mobile ground power stations, and in total energy units.
A permanent-magnet (PM) generator with surface-mounted magnets and a controlled bridge rectifier are used, because of their high efficiency, high reliability, high power density, and the possibilities for high speed.

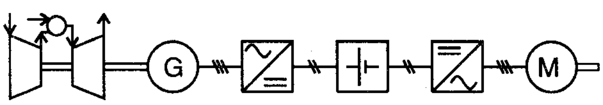

Figure 1: The drive system of a series-hybrid vehicle, consisting of gas turbine, PM generator, rectifier, accumulator, inverter and motor.

Objective of the paper. The aim of this paper is to introduce a model of a PM machine including the eddy-current loss in the magnets due to the time harmonics of the stator currents. This loss is represented by magnet loss resistances in the equivalent circuits. The model is verified by means of locked-rotor tests. To illustrate the usefulness of the model, it is used to calculate the loss in the magnets of a PM machine with rectifier load.
When designing high-speed PM machines, it is important to know the eddy-current loss in the magnets, because a high loss may result in demagnetization of the magnets. This loss heats the rotor, which can only difficultly be cooled, and the magnets demagnetize if they become too hot (NdFeB demagnetizes at about 120° C).
In literature, the eddy-current loss in the magnets is usually neglected. However, in high-speed machines, this loss may be problematic. Henneberger and Schleuter (1) report of high-speed PM machines that become too hot due to this loss. Van der Meer and Rietema (2) state that this loss is comparable to the iron loss in the solid rotor of their high-speed machine. Also the calculations in this paper affirm that this loss may be problematic.
Boules et al (3), Weschta (4), and Demel (5), also calculate the eddy-current loss in the magnets. However, they all replace the magnets by a cylinder of magnet material. This does not give realistic results when the magnets are divided into small magnet blocks to reduce the eddy-current loss. In this paper, the magnet blocks are not replaced by a cylinder of magnet material.
Sebastian and Slemon (6) also represented the eddy currents in the magnets by a resistance in the equivalent circuit. However, they used the model to study the transient performance, and not the eddy-current loss.

Main assumptions. The model derived in this paper is based on the following assumptions.
1) The magnetic flux density crosses the air gap and the magnets perpendicular: the field is one-dimensional.
2) The effect of the eddy currents in the magnets on the magnetic flux density is negligible.
3) The magnets are so small that the magnetic flux density can be considered constant over the magnet breadth. When large magnets are used, other methods (3), (4), (5) may give better results.
4) End effects are negligible. Therefore, the current density in the magnets only has an axial component.
5) Losses due to space harmonics of the stator windings and due to the slotting of the stator are neglected. However, calculations (5) indicate that these losses are smaller than the losses due to the time harmonics of the stator currents.
With these assumptions, the loss is not calculated very accurate, but a reliable approximation is obtained.

Structure of this paper. This paper starts with the calculation of the eddy-current loss in a PM machine. Next, it shows that this loss can be represented by a magnet loss resistance in the model. Subsequent, the derived model is verified by means of locked-rotor tests. As an example, the model is used to calculate the loss in the magnets of a PM machine with rectifier load. Finally, some conclusions are drawn.

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THE EDDY-CURRENT LOSS IN THE MAGNETS

Eddy-current loss in a magnet. Figure 2 depicts a cross-section of a magnet. The current density flows in the axial z-direction, the magnetic flux density is perpendicular to the plane of the drawing.

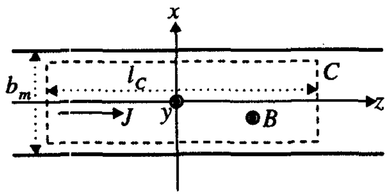

Figure 2: Cross-section of the magnet.

The current density in the magnets is calculated with the second of Maxwell's equations:

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{a} \qquad (1)$$

This equation is applied to the dashed closed path C in figure 2, which has length l_C in the z-direction. In this equation, it is used that

1) the electric field strength is the product of the current density and the resistivity of the magnet: $\vec{E} = \rho_m \vec{J}$,
2) end effects are assumed to be negligible, so that the two sides of this closed path parallel to the x-axis do not contribute to the line integral,
3) the magnetic flux density is assumed to be constant over the magnet breadth b_m, and
4) consequentially, the current density is an odd function of x: $J(-x) = -J(x)$.

The resulting expression for the current density is

$$J(x) = \frac{x}{\rho_m}\frac{dB}{dt} \qquad (2)$$

With this, the eddy-current loss per unit of magnet volume is calculated as

$$k_m = \frac{1}{b_m}\int_{-b_m/2}^{b_m/2} \rho_m J^2(x)\,dx = \frac{b_m^2}{12\rho_m}\left(\frac{dB}{dt}\right)^2 \qquad (3)$$

This shows that decreasing the magnet breadth b_m is an effective means to decrease the loss in the magnets.

Eddy-current loss in the PM machine. Losses due to space harmonics of the magnetic flux density are neglected. Therefore, the magnetic flux density causing the losses can be written as

$$B(\alpha_r) = \hat{B}\cos(p(\alpha_r - \beta)) \qquad (4)$$

where p is the number of pole pairs of the machine.
In this equation, both $\hat{B}$ and β may be functions of time. The magnets are numbered 1 to N_m, and the axis of kth magnet lays at rotor coordinate α_k, see figure 3.
The magnetic flux density was assumed to be constant over the magnet breadth b_m. With this, the magnetic flux density in the kth magnet can be written as

$$B_k = B(\alpha_k) = \hat{B}\cos(p(\alpha_k - \beta)) \qquad (5)$$

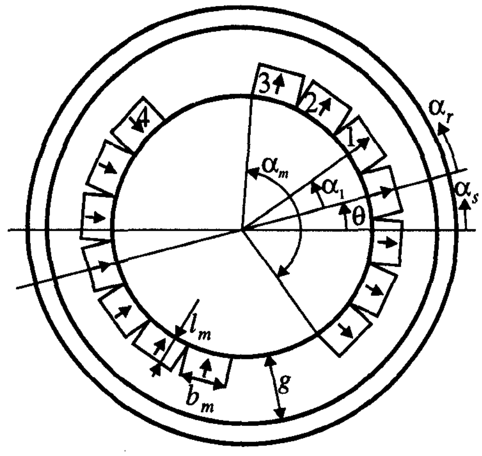

Figure 3: Cross-section of a two-pole PM machine.

When this magnetic flux density is used in equation (3), the eddy-current loss per unit of magnet volume in the kth magnet is calculated as

$$k_{m,k} = \frac{b_m^2}{12\rho_m}\left(\frac{d}{dt}\{\hat{B}\cos(p(\alpha_k - \beta))\}\right)^2 \qquad (6)$$

Multiplication of this expression by the volume of the magnet gives the eddy-current loss in the kth magnet. Summation over all magnets results in the total eddy-current loss in the magnets:

$$\begin{aligned} P_m &= l l_m b_m \sum_{k=1}^{N_m} \frac{b_m^2}{12\rho_m}\left(\frac{d}{dt}\{\hat{B}\cos(p(\alpha_k - \beta))\}\right)^2 \\ &\approx 2prll_m \frac{b_m^2}{12\rho_m}\int_{-\alpha_m/2}^{\alpha_m/2}\left(\frac{d}{dt}\{\hat{B}\cos(p(\alpha_r - \beta))\}\right)^2 d\alpha_r \\ &= \frac{rll_m b_m^2}{12\rho_m}\left\{(p\alpha_m + \sin(p\alpha_m))\left(\frac{d}{dt}\{\hat{B}\cos(p\beta)\}\right)^2 \right. \\ &\qquad \left. + (p\alpha_m - \sin(p\alpha_m))\left(\frac{d}{dt}\{\hat{B}\sin(p\beta)\}\right)^2\right\} \end{aligned} \qquad (7)$$

where l_m is the thickness of the magnets, α_m is the magnet pole arc (see figure 3), r is the air-gap radius, and l is the axial length of the machine.
The approximation in this equation is based on the assumption that the magnet breadth b_m is small.

THE MAGNET LOSS RESISTANCE

To represent the eddy-current loss in the magnets by magnet loss resistances in equivalent circuits, we need the voltage equations of the machine. Here, a condensed derivation of these voltage equations is given; for a more thorough derivation is referred to Slemon (7).

The magnetic flux density due to the stator currents. Because the losses due to space harmonics of the magnetic flux density are neglected, only the fundamental of the stator winding distribution is considered. The conductor density n_{sa} (the number of conductors per radian) of the fundamental of the winding distribution of phase a is given by

$$n_{sa}(\alpha_s) = \tfrac{1}{2}N_s \sin(p\alpha_s) \tag{8}$$

where N_s is the number of turns of the fundamental of the stator winding distribution, which is related to the actual number of turns N by

$$N_s = \frac{4}{\pi}k_w N \tag{9}$$

where k_w is the winding factor of the actual winding. When a current i_{sa} flows in this winding, the magnetic flux density in the air gap with length g (see figure 3) can be calculated as

$$B_{sa}(\alpha_s) = \frac{\mu_0 N_s}{2gp} i_{sa}\cos(p\alpha_s) \tag{10}$$

In the same way, the magnetic flux density due to the other phases can be calculated:

$$B_s(\alpha_s) = \frac{\mu_0 N_s}{2gp}\Big\{ i_{sa}\cos(p\alpha_s) + i_{sb}\cos(p\alpha_s - \tfrac{2}{3}\pi) + i_{sc}\cos(p\alpha_s - \tfrac{4}{3}\pi)\Big\} \tag{11}$$

The fluxes linked by the stator windings. At an arbitrary moment, the magnetic flux density in the air gap can be written as a Fourier series:

$$B(\alpha_s) = \sum_{k=1,3,5,..}^{\infty} \hat{B}_k \cos(kp(\alpha_s - \beta_k)) \tag{12}$$

To obtain an expression for the flux linked by a stator winding, first the flux $\psi_t(\alpha')$ linked by a full-pitch turn at stator coordinate α' is calculated. The flux linkage of this turn is given by

$$\psi_t(\alpha') = \iint_S \vec{B}\cdot d\vec{a} = \int_{\alpha'-\pi/p}^{\alpha'} B(\alpha_s) lr\, d\alpha_s \tag{13}$$

Using the equations (8), (12), and (13), the flux ψ_{sa} linked by stator phase a is obtained by integration:

$$\psi_{sa} = p\int_0^{\pi/p} n_{sa}(\alpha')\,\psi_t(\alpha')\,d\alpha' = \frac{\pi r l N_s}{2p}\hat{B}_1\cos(p\beta_1) \tag{14}$$

In the same way, the fluxes linked by the other stator phases can be calculated:

$$\vec{\psi}_s = \begin{bmatrix}\psi_{sa}\\ \psi_{sb}\\ \psi_{sc}\end{bmatrix} = \frac{\pi r l N_s}{2p}\hat{B}_1 \begin{bmatrix}\cos(p\beta_1)\\ \cos(p\beta_1 - \tfrac{2}{3}\pi)\\ \cos(p\beta_1 - \tfrac{4}{3}\pi)\end{bmatrix} \tag{15}$$

The voltage equations. When the resistance of a stator phase is called R_s, the stator voltages can be written as

$$\vec{u}_s = R_s\vec{i}_s + \frac{d\vec{\psi}_s}{dt}\ ; \qquad \vec{u}_s = \begin{bmatrix}u_{sa}\\ u_{sb}\\ u_{sc}\end{bmatrix} ; \qquad \vec{i}_s = \begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{sc}\end{bmatrix} \tag{16}$$

The flux linkage of this equation is separated into different contributions, namely:

1) the flux linkage due to leakage $\vec{\psi}_{s\sigma}$, and
2) the flux linkage due to the air-gap field, which again is separated into

a) the flux linkage due to the magnets $\vec{\psi}_{sm}$, and

b) the flux linkage due to the stator currents $\vec{\psi}_{ss}$.

The self-inductances of the leakage flux of the stator windings are called $L_\sigma + M_{s\sigma ab}$, the mutual inductances of the leakage flux between the different stator phases are called $M_{s\sigma ab}$. Hence, the leakage flux can be written as

$$\vec{\psi}_{s\sigma} = \begin{bmatrix} L_\sigma + M_{s\sigma ab} & M_{s\sigma ab} & M_{s\sigma ab}\\ M_{s\sigma ab} & L_\sigma + M_{s\sigma ab} & M_{s\sigma ab}\\ M_{s\sigma ab} & M_{s\sigma ab} & L_\sigma + M_{s\sigma ab}\end{bmatrix}\vec{i}_s = \boldsymbol{L}_{s\sigma}\vec{i}_s \tag{17}$$

The magnetization of the magnets is constant. Therefore, the flux linkage due to the magnets $\vec{\psi}_{sm}$ only depends on the rotor position angle θ (see figure 3). The time derivative of this flux linkage is the no-load voltage:

$$\vec{e}_p = \frac{d\vec{\psi}_{sm}}{dt} \tag{18}$$

The flux linkages of the stator windings due to the flux density of the three-phase stator (equation (11)) are calculated in the same way as in equation (15) as

$$\vec{\psi}_{ss} = \boldsymbol{L}_{ss}\vec{i}_s\ ; \qquad \boldsymbol{L}_{ss} = \frac{1}{3}L_m\begin{bmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{bmatrix} ; \qquad L_m = \frac{3\mu_0 \pi r l N_s^2}{8gp^2} \tag{19}$$

With this, the voltage equation can be written as

$$\vec{u}_s = \vec{e}_p + R_s\vec{i}_s + (\boldsymbol{L}_{s\sigma} + \boldsymbol{L}_{ss})\frac{d\vec{i}_s}{dt} \tag{20}$$

By means of the well-known Park transformation, this voltage equation is transformed to the dq-system:

$$\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} = \boldsymbol{P}\vec{i}_s\ ; \quad \begin{bmatrix}u_{sd}\\ u_{sq}\end{bmatrix} = \boldsymbol{P}\vec{u}_s\ ; \quad \begin{bmatrix}\psi_{sd}\\ \psi_{sq}\end{bmatrix} = \boldsymbol{P}\vec{\psi}_s\ ; \quad \begin{bmatrix}0\\ e_{pq}\end{bmatrix} = \boldsymbol{P}\vec{e}_p\ ;$$
$$\boldsymbol{P} = \sqrt{\frac{2}{3}}\begin{bmatrix}\cos(p\theta) & \cos(p\theta - \tfrac{2}{3}\pi) & \cos(p\theta - \tfrac{4}{3}\pi)\\ -\sin(p\theta) & -\sin(p\theta - \tfrac{2}{3}\pi) & -\sin(p\theta - \tfrac{4}{3}\pi)\end{bmatrix} \tag{21}$$

The zero-component was omitted, because it is zero. When this transformation is applied to the voltage equations (equation (20)), the result is

$$\begin{bmatrix}u_{sd}\\ u_{sq}\end{bmatrix} = \begin{bmatrix}0\\ e_{pq}\end{bmatrix} + R_s\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + (L_\sigma + L_m)\frac{d}{dt}\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + p\Omega L_m\begin{bmatrix}-i_{sq}\\ i_{sd}\end{bmatrix} \tag{22}$$

where Ω is the mechanical angular speed of the rotor.

The magnet loss resistance. Figure 4 depicts the equivalent circuits of the PM machine with the magnet loss resistances. To examine if the eddy-current loss in

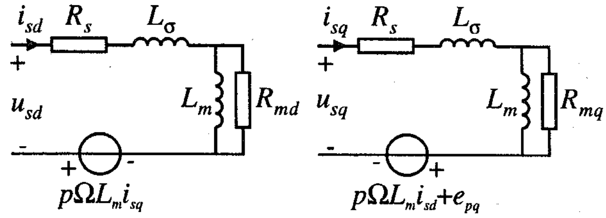

Figure 4: The direct-axis and the quadrature-axis equivalent circuits of the PM machine.

the magnets can be represented by these magnet loss resistances, the loss in these resistances due to the magnetic flux density of equation (4) is calculated and compared to the loss calculated in equation (7).
The flux linkages of the stator windings due to the magnetic flux density of equation (4) are calculated in the same way as in equation (15). They are transformed to the dq-system with the Park transformation (equation (21)):

$$\begin{bmatrix}\psi_{sd}\\ \psi_{sq}\end{bmatrix} = \sqrt{\frac{3}{2}}\frac{\pi r l N_s}{2p}\hat{B}\begin{bmatrix}\cos(p\beta)\\ \sin(p\beta)\end{bmatrix} \tag{23}$$

The voltages induced in the inductances L_m are the time derivatives of these fluxes. Therefore, the losses in the magnet loss resistances R_{md} and R_{mq} are given by

$$P_m = \frac{1}{R_{md}}\left(\frac{\mathrm{d}\psi_{sd}}{\mathrm{d}t}\right)^2 + \frac{1}{R_{mq}}\left(\frac{\mathrm{d}\psi_{sq}}{\mathrm{d}t}\right)^2 \tag{24}$$

Comparison of this equation with equation (7) shows that the losses indeed can be represented by magnet loss resistances if the values of these resistances are given by

$$R_{md} = \frac{9\rho_m \pi^2 r l N_s^2}{2 l_m p^2 b_m^2 (p\alpha_m + \sin(p\alpha_m))}$$
$$R_{mq} = \frac{9\rho_m \pi^2 r l N_s^2}{2 l_m p^2 b_m^2 (p\alpha_m - \sin(p\alpha_m))} \tag{25}$$

It should be noticed that when sinusoidal stator currents are used, the loss in the magnets is zero.

LOCKED-ROTOR TESTS

The derived model is verified by comparing the calculated and the measured impedances of a PM machine with locked rotor. The rotor of the used PM machine is completely covered with small magnets, so there is hardly any difference between the direct-axis and the quadrature-axis impedances. Therefore, only direct-axis locked-rotor tests ($p\theta = \pi/2$) are reported.
During the locked-rotor tests, a sinusoidal voltage is supplied to the phases b and c, which are connected in series, as depicted in figure 5. Therefore, $i_{sb} = i$, $i_{sc} = -i$, and $u_{sb} - u_{sc} = u$ are valid. With the Park transformation (equation (21)), the dq-components of the currents and voltages are calculated. The direct-axis impedance $\underline{Z}_d$ during the locked-rotor tests is calculated as

$$\underline{Z}_d = \frac{\hat{\underline{u}}}{\hat{\underline{i}}} = 2\frac{\hat{\underline{u}}_{sd}}{\hat{\underline{i}}_{sd}} = 2\left\{R_s + \mathrm{j}\omega L_\sigma + \frac{\mathrm{j}\omega L_m R_{md}}{R_{md} + \mathrm{j}\omega L_m}\right\} \tag{26}$$

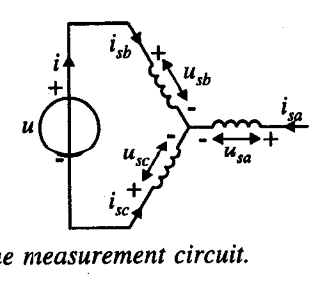

Figure 5: The measurement circuit.

The resistance and inductance are calculated from this impedance by

$$R_d = \mathrm{Re}(\underline{Z}_d)\ ; \quad L_d = \frac{1}{\omega}\mathrm{Im}(\underline{Z}_d) \tag{27}$$

The resistance and the inductance of the PM machine are determined from the measured voltage U over the terminals, the current I flowing through the phases, and the power P dissipated in the machine as

$$R = \frac{P}{I^2}\ ; \quad L = \frac{1}{\omega}\sqrt{\left(\frac{U}{I}\right)^2 - R^2} \tag{28}$$

Figure 6 depicts the measured and the calculated inductance and resistance in the direct axis.
Differences between the measured and the calculated resistance are probably caused by the neglect of the iron loss, which is subject of further research.
The model was derived on the assumption that the effect of the eddy currents on the magnetic flux density was negligible. Above 10 kHz, this assumption is not valid, as appears from the decrease of the inductance. However, the agreement between measurements and calculations remains reasonable.

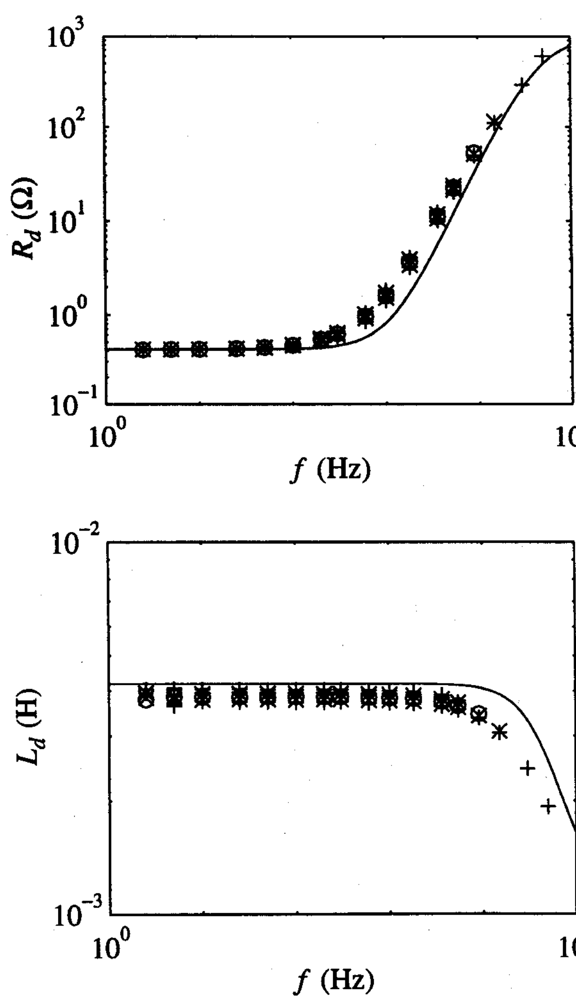

Figure 6: Measured (+:I=0.2A, ×:I=0.5A, ○:I=1A, ∗:I=2A, +:I=5A) and calculated (—) resistance and inductance in the direct axis.

The reasonable agreement between measurements and calculations shows that the proposed model is useful.

A PM MACHINE WITH RECTIFIER LOAD

To illustrate the usefulness of the proposed model, the PM machine that was also used for the locked-rotor tests, is loaded with a controlled bridge rectifier. The steady-state performance is calculated with the calculation method described by Polinder et al (8).
Figure 7 depicts the measured and the calculated line voltage and phase current which agree very well. Here, the speed of the rotor n is 6,000 rpm.
In table 1, the calculated loss P_m in the magnets is given. For comparison, the stator copper loss P_{Cu} and the generated power P_{gen} are also calculated and given.
Also the calculated powers at 30,000 rpm with the same firing angle and the same current in the direct-current circuit are given. This shows that at high speeds the eddy-current loss in the magnets may be a serious problem, because it is about proportional to the square of the frequency, as also follows from equation (3).

TABLE 1: Losses in a PM machine at different speeds.

n (rpm)	P_m (W)	P_{Cu} (W)	P_{gen} (kW)
6,000	11	160	8,0
30,000	254	160	40

CONCLUSION

When designing high-speed PM machines, it is important to know the eddy-current loss in the magnets, because it may be very large. Therefore, this paper introduces a model of a PM machine including the eddy-current loss in the magnets due to the time harmonics of the stator currents. It shows that these losses can be represented by magnet loss resistances in the equivalent circuits. The model is verified by means of locked-rotor tests. The loss in the magnets can be decreased by using smaller magnets or by using sinusoidal stator currents.

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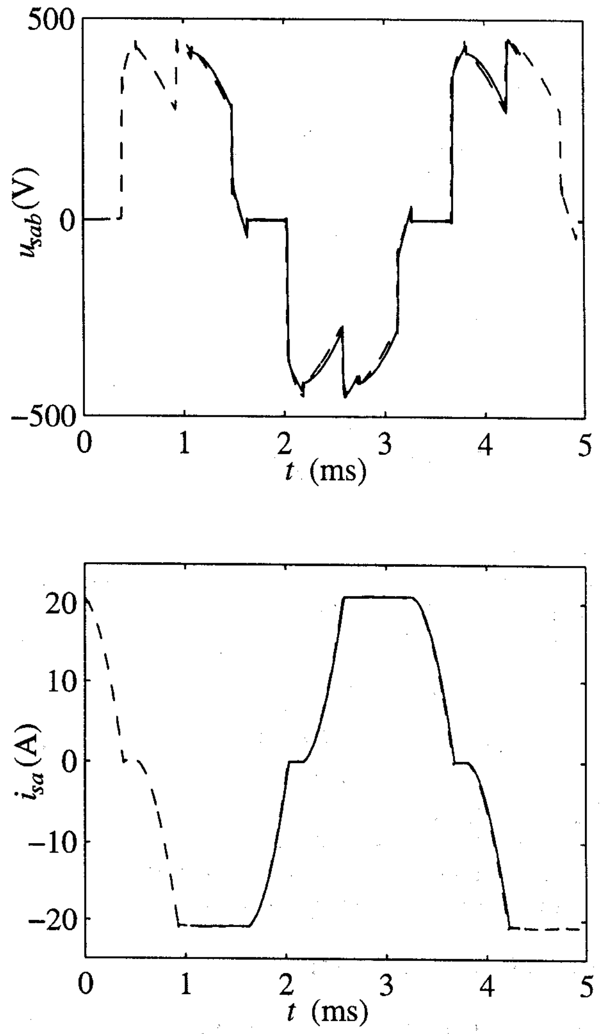

Figure 7: Measured (--) and calculated (—) line voltage and phase current.

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