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A SIMPLE MODEL OF A SYNCHRONOUS MACHINE WITH DIODE RECTIFIER
USING STATE VARIABLES

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Most simulation methods for a synchronous machine with rectifier require a lot of computation time. Neglecting the harmonics on the phase currents and the ripple on the direct current, a less detailed method based on state description is derived. The commutation phenomena in the rectifier are taken into account.

1. INTRODUCTION

The interest in renewable energy has resulted in much research in wind energy conversion systems. One of the favourite conversion systems is the series system synchronous machine - diode rectifier - smoothing coil - inverter as depicted in figure 1, by means of which variable-speed operation of the wind turbine is possible, so that wind energy as well as system components may be utilized in an optimal way [1].

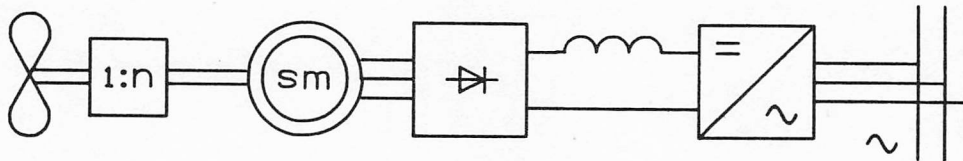


Figure 1 A wind-energy conversion-system with synchronous machine and dc-link

Although the steady-state behaviour of this system is good, its transient behaviour may be problematic, especially with a large system. In order to investigate the dynamic behaviour of this system, a detailed simulation of the system is often used. Such a simulation, in which for example the commutation in the rectifier can be recognized, requires a lot of computation time [2]. However, in order to investigate the stability, not only the electrical part, but the whole wind energy conversion system, so including the mechanical part, has to be considered. Using a detailed simulation of the synchronous machine with rectifier, the computation time would be too large for normal use.

Within the framework of the Netherlands Wind Energy Research Program, the group Electromechanics and Power Electronics of the Eindhoven University of Technology received the research request to find a less detailed simulation method. In this paper, the principles of the method found are presented.

The system considered here is given in figure 2. First, the two important parts of this system, the synchronous machine and the rectifier, will be described separately. Next, the steady-state model of the system is given. Using the suppositions from this model, the dynamic model is derived.

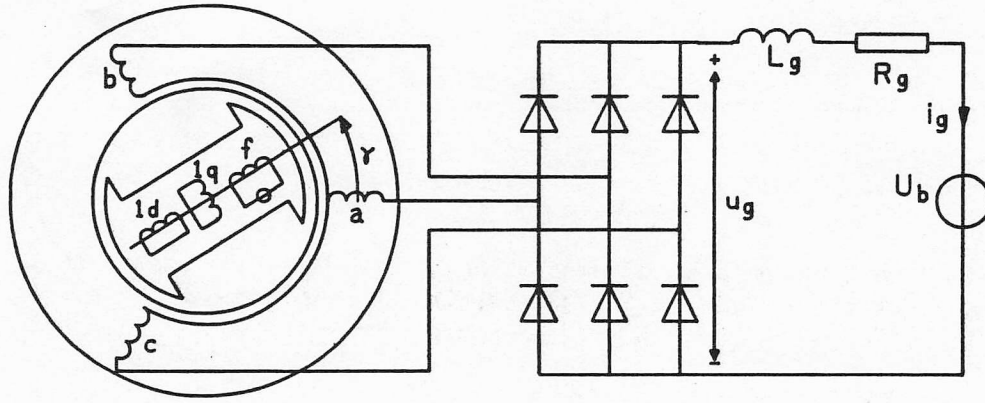


Figure 2 The system considered here

2. THE THREE-PHASE DIODE-BRIDGE RECTIFIER

In the description of the rectifier, the circuit shown in figure 3 will be used. The rectifier is fed by a three-phase voltage source with internal self-inductance L_c and internal voltages e_a , e_b and e_c according to

$$e_a = \hat{e} \cos(\omega t) ; \quad e_b = \hat{e} \cos(\omega t - \frac{2}{3}\pi) ; \quad e_c = \hat{e} \cos(\omega t - \frac{4}{3}\pi) \quad (2.1)$$

where ω is a constant angular frequency and \hat{e} is a constant amplitude. The rectifier is loaded by a constant current source I_g . The diodes will be considered as ideal switches; resistances in the circuit are neglected. Thanks to the symmetry of the circuit and of the currents and voltages in this circuit, the description of the rectifier can be restricted to an interval of $\pi/3$ rad of length. Here the interval between the angle corresponding to the starting instant of diode D_1 and the angle corresponding to the starting instant of diode D_6 will be used. Diode D_1 will turn on at the instant at which the voltage e_a reaches the same (positive) value as the voltage e_c : $\omega t = -\pi/3$. Hence, the considered interval, which is indicated by means of a thick line piece in figure 4, is given by $-\pi/3 < \omega t < 0$. The angle of overlap μ , which will be defined later on, is supposed to be smaller than $\pi/3$ rad.

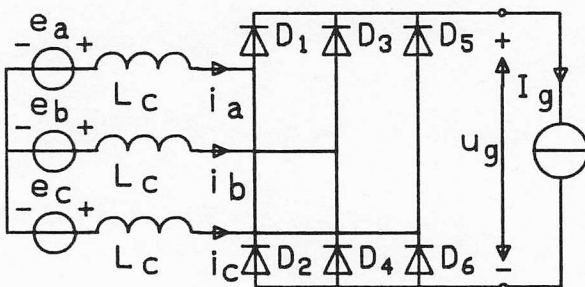


Figure 3 Base circuit for the rectifier description

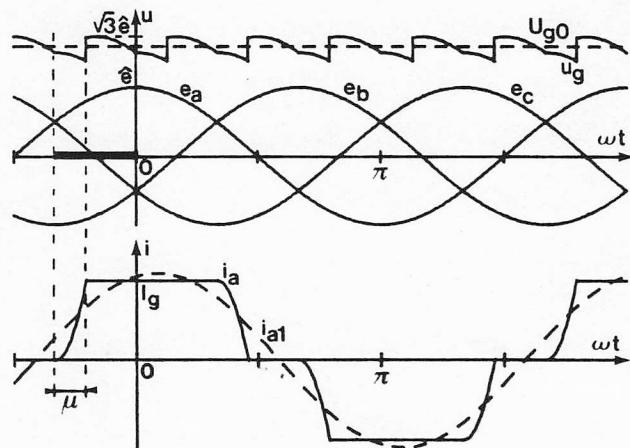


Figure 4 Some quantities as functions of ωt ($\mu = 0.4$)

Just before the considered interval, the diodes D_4 and D_5 are conducting; at the beginning of this interval diode D_1 will turn on and the current I_g starts to transfer from diode D_5 to diode D_1 (the starting instant of the commutation). During this commutation only the diodes D_1 , D_4 and D_5 are conducting. Hence, using (2.1) and the initial condition $i_a(-\pi/3) = 0$, the following relations can be given for the commutation interval considered:

$$\begin{aligned} i_a &= \frac{\sqrt{3}\hat{e}}{2\omega L_c} \{1 - \cos(\omega t + \frac{\pi}{3})\} ; & i_b &= -I_g \\ i_c &= I_g - \frac{\sqrt{3}\hat{e}}{2\omega L_c} \{1 - \cos(\omega t + \frac{\pi}{3})\} ; & u_g &= \frac{3}{2}\hat{e} \cos(\omega t + \frac{\pi}{3}) \end{aligned} \quad (2.2)$$

The commutation is finished when the current through diode D_5 (i_c) becomes zero. The time expressed in angular measure, elapsed from the beginning of the commutation until the end of the commutation is called the angle of overlap μ . In the considered interval the commutation is finished at the instant corresponding to $\omega t = -\pi/3 + \mu$. From the condition $i_c(-\pi/3 + \mu) = 0$ and (2.2), it follows:

$$1 - \cos \mu = \frac{2\omega L_c I_g}{\sqrt{3}\hat{e}} \quad (2.3)$$

After the commutation being finished, only the diodes D_1 and D_4 are conducting. Using figure 3 and the voltage expressions (2.1), the following expressions can be given (only valid in the second part of the interval considered):

$$i_a = -i_b = I_g ; \quad i_c = 0 ; \quad u_g = e_a - e_b = \sqrt{3}\hat{e} \cos(\omega t + \frac{\pi}{6}) \quad (2.4)$$

The average value of the voltage u_g can be found by means of the expressions (2.2), (2.4) and (2.3):

$$U_{g0} = \frac{3}{\pi} \int_{-\frac{\pi}{3}}^0 u_g d\omega t = \frac{3}{\pi} \sqrt{3}\hat{e} - \frac{3}{\pi} \omega L_c I_g \quad (2.5)$$

By means of Fourier analysis and the equations (2.2), (2.3) and (2.4), the fundamental component of the phase current may be expressed as

$$i_{a1}(\omega t) = i_{act} \cos(\omega t) + i_{rea} \sin(\omega t) \quad (2.6a)$$

where the active and the reactive component coefficients are given by

$$i_{act} = \frac{\sqrt{3}}{\pi} I_g (1 + \cos \mu) ; \quad i_{rea} = \frac{3\hat{e}}{4\omega L_c \pi} \{2\mu - \sin(2\mu)\} \quad (2.6b)$$

In many practical situations, the ripple on the direct current may be neglected, so that the description in the previous part may be used for the steady state. The description may also be used for slow changes in the amplitude or frequency of the phase voltages and the average value of the direct current.

The dynamic model introduced in this way may be improved by enlarging the inductance in the dc-circuit with $2L_c$ [3]. This enlargement corresponds with the inductance seen from the dc-side of the rectifier when two diodes are conducting. Using (2.5), the equivalent circuit given in figure 5 may be composed.

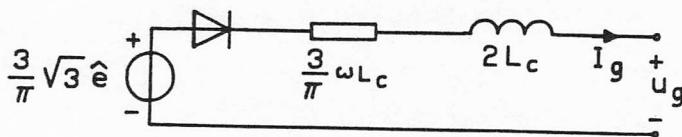


Figure 5 An equivalent circuit for the rectifier

3. THE SYNCHRONOUS MACHINE

As in many cases, in this paper the (salient pole) synchronous machine is represented with one damper winding on the direct axis and one damper winding on the quadrature axis. In order to describe this machine, the Park transformation for the phase current according to

$$i_d = \sqrt{\frac{2}{3}} \{i_a \cos \gamma + i_b \cos(\gamma - \frac{2}{3}\pi) + i_c \cos(\gamma - \frac{4}{3}\pi)\} \quad (3.1a)$$

$$i_q = \sqrt{\frac{2}{3}} \{i_a \sin \gamma + i_b \sin(\gamma - \frac{2}{3}\pi) + i_c \sin(\gamma - \frac{4}{3}\pi)\} \quad (3.1b)$$

$$i_0 = \sqrt{\frac{1}{3}} \{i_a + i_b + i_c\} \quad (3.1c)$$

will be used. The angle γ is defined in figure 2. For the phase voltages and flux linkages similar formulas will be used. As may be seen in figure 2, the homopolar current is zero. For that reason no attention is paid to this component. With the usual suppositions for synchronous machines (see for example [4]), the stator voltage equations may be given by

$$u_q = -R_a i_q - \frac{d\psi_q}{dt} + \omega \psi_d; \quad u_d = -R_a i_d - \frac{d\psi_d}{dt} - \omega \psi_q \quad (3.2)$$

where ω is the angular speed of the rotor ($\omega = \frac{d\gamma}{dt}$). The rotor voltages are

$$u_f = R_f i_f + \frac{d\psi_f}{dt}; \quad 0 = R_{11d} i_{1d} + \frac{d\psi_{1d}}{dt}; \quad 0 = R_{11q} i_{1q} + \frac{d\psi_{1q}}{dt} \quad (3.3)$$

In these expressions the subscripts 1d, f and 1q refer to, respectively, the damper winding on the direct axis, the excitation (field) winding and the damper winding on the quadrature axis.

The flux linkages in the voltage equations are given by

$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_{1d} \end{bmatrix} = \begin{bmatrix} L_d & L_{afd} & L_{a1d} \\ L_{afd} & L_{ffd} & L_{f1d} \\ L_{a1d} & L_{f1d} & L_{11d} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{1d} \end{bmatrix}; \quad \begin{bmatrix} \psi_q \\ \psi_{1q} \end{bmatrix} = \begin{bmatrix} L_q & L_{a1q} \\ L_{a1q} & L_{11q} \end{bmatrix} \begin{bmatrix} i_q \\ i_{1q} \end{bmatrix} \quad (3.4)$$

Substituting the expressions (3.4) into the voltage equations (3.2) and (3.3) results in a set of two equations for the quadrature axis

$$-u_q = -\omega(L_d i_d + L_{afd} i_f + L_{a1d} i_{1d}) + R_a i_q + L_q \frac{di_q}{dt} + L_{a1q} \frac{di_{1q}}{dt} \quad (3.5a)$$

$$0 = R_{11q} i_{1q} + L_{a1q} \frac{di_q}{dt} + L_{11q} \frac{di_{1q}}{dt} \quad (3.5b)$$

and a set of three equations for the direct axis

$$-u_d = \omega(L_q i_q + L_{a1q} i_{1q}) + R_a i_d + L_d \frac{di_d}{dt} + L_{afd} \frac{di_f}{dt} + L_{a1d} \frac{di_{1d}}{dt} \quad (3.6a)$$

$$u_f = R_f i_f + L_{afd} \frac{di_d}{dt} + L_{ffd} \frac{di_f}{dt} + L_{f1d} \frac{di_{1d}}{dt} \quad (3.6b)$$

$$0 = R_{11d} i_{1d} + L_{a1d} \frac{di_d}{dt} + L_{f1d} \frac{di_f}{dt} + L_{11d} \frac{di_{1d}}{dt} \quad (3.6c)$$

The per-unit system will not be used in this paper, so that the base values for voltage, current and time for the stationary circuit are,

respectively, 1V, 1A, and 1s. However, in order to get equivalent circuits suitable for simulation, the rotor base values for voltage and for current are chosen different. This is realized by introducing three new currents: one for the quadrature-axis damper winding, one for the direct-axis damper winding, and one for the excitation winding

$$i_Q = \frac{1}{C_Q} i_{1q}; \quad i_D = \frac{1}{C_D} i_{1d}; \quad i_F = \frac{1}{C_F} i_f \quad (3.7a)$$

$$\text{where } C_Q = \frac{L_{a1q}}{L_{11q}}; \quad C_F = \frac{L_{afd}L_{11d} - L_{a1d}L_{f1d}}{L_{ffd}L_{11d} - L_{f1d}^2}; \quad C_D = \frac{L_{a1d}L_{ffd} - L_{afd}L_{f1d}}{L_{ffd}L_{11d} - L_{f1d}^2} \quad (3.7b)$$

The motivation of this choice may be found in [2]. In order to get simple equations, the following parameters are introduced:

$$L_{qQ} = C_Q L_{a1q}; \quad L_q'' = L_q - L_{qQ}; \quad R_Q = C_Q^2 R_{11q}; \quad L_{dF} = C_F L_{afd}; \quad L_{dD} = C_D L_{a1d}; \\ L_d'' = L_d - L_{dF} - L_{dD}; \quad L_{FD} = -C_F C_D L_{f1d}; \quad R_F = C_F^2 R_f; \quad u_F = C_F u_f \quad (3.8)$$

Using (3.7) and (3.8) and multiplying (3.5b) with C_Q , the set of equations (3.5) becomes

$$-u_q = -\omega(L_d'' i_d + L_{dF} i_F + L_{dD} i_D) + R_a i_q + L_q'' \frac{di_q}{dt} + L_{qQ} \left(\frac{di_q}{dt} + \frac{di_Q}{dt} \right) \quad (3.9a)$$

$$0 = R_Q i_Q + L_{qQ} \left(\frac{di_q}{dt} + \frac{di_Q}{dt} \right) \quad (3.9b)$$

The equivalent circuit shown in figure 6a is given by these voltage equations.

Using (3.7) and (3.8), multiplying (3.6b) with C_F , and multiplying (3.6c) with C_D , the set of equations (3.6) becomes

$$-u_d = \omega(L_q i_q + L_{qQ} i_Q) + R_a i_d + L_d \frac{di_d}{dt} + L_{dF} \frac{di_F}{dt} + L_{dD} \frac{di_D}{dt} \\ u_F = R_F i_F + L_{dF} \frac{di_d}{dt} + (L_{dF} + L_{FD}) \frac{di_F}{dt} - L_{FD} \frac{di_D}{dt} \quad (3.10) \\ 0 = R_D i_D + L_{dD} \frac{di_d}{dt} - L_{FD} \frac{di_F}{dt} + (L_{dD} + L_{FD}) \frac{di_D}{dt}$$

The equivalent circuit shown in figure 6b is given by these voltage equations.

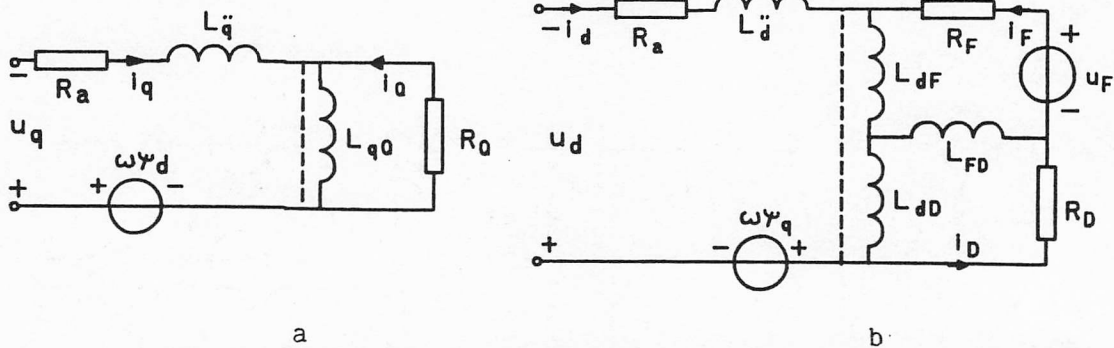


Figure 6 The quadrature-axis (a) and direct-axis (b) equivalent circuits.

As may be seen in figure 6, L_q'' and L_d'' are, respectively, the normal quadrature-axis and the normal direct-axis sub-transient inductances.

4. THE STEADY-STATE MODEL OF THE SYNCHRONOUS MACHINE WITH DIODE BRIDGE

When a diode bridge is connected to a synchronous machine, it can be proven that the direct-axis and the quadrature-axis components of the armature currents consist of a constant part (I_d and I_q) and a fast changing part which may be represented by a Fourier series with terms with an angular frequency which is a multiple of 6ω [5]. For these fast changing current components the network parts to the right of the dashed lines in figure 6 resemble short circuits. This resemblance is supposed to be exact, so that the dashed lines can be interpreted as short circuits for these components.

In order to combine the models described in the chapters 2 and 3, the equivalent circuit of the synchronous machine should correspond with figure 3. When $L_d'' = L_q'' = L''$ this is easily achieved by rearranging the equivalent circuit of the synchronous machine. In most other cases ($L_d'' \neq L_q''$) an approximation can be used [5].

Using the Park transformation according to (3.1) for the voltages with $u_0 = 0$, the stator voltages may be given by

$$u_a = \sqrt{\frac{2}{3}} \{u_d \cos \gamma + u_q \sin \gamma\} \quad (4.1a)$$

$$u_b = \sqrt{\frac{2}{3}} \{u_d \cos(\gamma - \frac{2}{3}\pi) + u_q \sin(\gamma - \frac{2}{3}\pi)\} \quad (4.1b)$$

$$u_c = \sqrt{\frac{2}{3}} \{u_d \cos(\gamma - \frac{4}{3}\pi) + u_q \sin(\gamma - \frac{4}{3}\pi)\} \quad (4.1c)$$

$$\text{Next, the internal voltages } e_d = u_d + R_a i_d + L'' \frac{di_d}{dt} + \omega L'' i_q \quad (4.2a)$$

$$e_q = u_q + R_a i_q + L'' \frac{di_q}{dt} - \omega L'' i_d \quad (4.2b)$$

are introduced. Using (3.1), (4.2), $i_0 = 0$, and $\gamma = \omega t + \pi/2$, and neglecting the armature resistance, the stator voltages according to (4.1) become

$$u_a = \sqrt{\frac{2}{3}} \{e_d \cos \gamma + e_q \sin \gamma\} - L'' \frac{di_a}{dt} = \hat{e} \cos(\omega t - \epsilon) - L'' \frac{di_a}{dt} \quad (4.3a)$$

$$u_b = \sqrt{\frac{2}{3}} \{e_d \cos(\gamma - \frac{2}{3}\pi) + e_q \sin(\gamma - \frac{2}{3}\pi)\} - L'' \frac{di_b}{dt} = \hat{e} \cos(\omega t - \epsilon - \frac{2}{3}\pi) - L'' \frac{di_b}{dt} \quad (4.3b)$$

$$u_c = \sqrt{\frac{2}{3}} \{e_d \cos(\gamma - \frac{4}{3}\pi) + e_q \sin(\gamma - \frac{4}{3}\pi)\} - L'' \frac{di_c}{dt} = \hat{e} \cos(\omega t - \epsilon - \frac{4}{3}\pi) - L'' \frac{di_c}{dt} \quad (4.3c)$$

$$\text{where } \hat{e} = \sqrt{\frac{2}{3}} \sqrt{e_q^2 + e_d^2}; \quad \epsilon = -\arctan\left(\frac{e_d}{e_q}\right) \quad (4.4)$$

After all these manipulations, the circuit given in figure 7 arises.

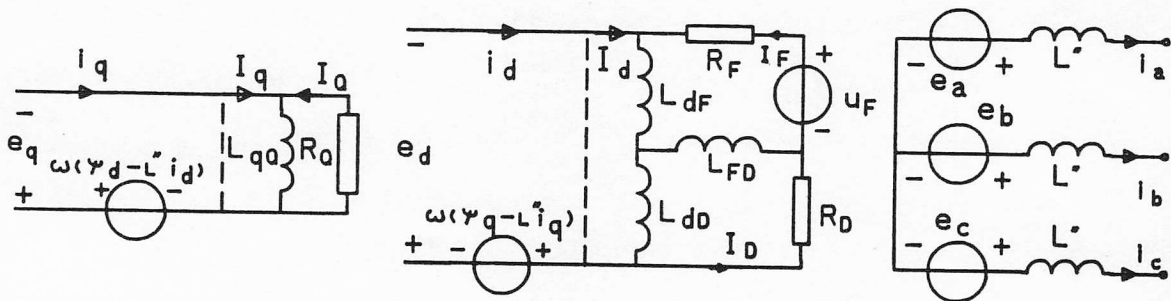


Figure 7 Equivalent circuit of the synchronous machine with $L'' = L_d'' = L_q''$

Since the dashed lines in this figure are supposed to be short circuits for the fast changing parts in i_q and i_d , e_q and e_d are constant. Hence, the voltages in the right part of figure 7 are sinusoidal, so that this part corresponds with figure 3; there is only a phase shift ϵ . The quadrature-axis and the direct-axis circuits in figure 7 may represent a normal synchronous machine with $R_a = 0$ and $L_d'' = L_q'' = 0$. This is called the "internal" machine. When this machine is extended by means of three self-inductances L'' , the original model machine arises (see figure 8). The phase angle ϵ represents the load angle of the "internal" machine.

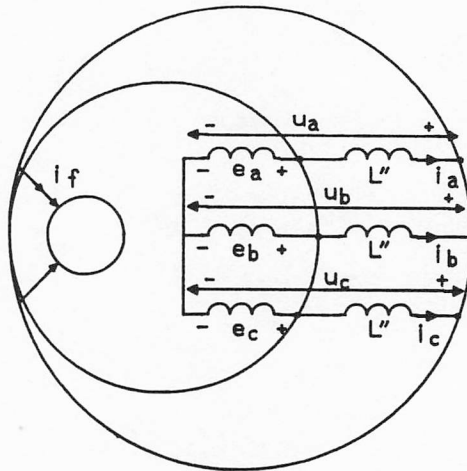


Figure 8 The "internal" machine

In order to compute the steady-state of the synchronous machine with rectifier, the constant parts of i_q and i_d , I_q and I_d , have to be known. These constant parts correspond with the basic harmonics of the armature phase currents ((2.6)), so that according to (3.1) ($\gamma = \omega t + \pi/2$):

$$I_q = \sqrt{\frac{3}{2}} (i_{act} \cos \epsilon - i_{rea} \sin \epsilon) ; \quad I_d = -\sqrt{\frac{3}{2}} (i_{act} \sin \epsilon + i_{rea} \cos \epsilon) \quad (4.5)$$

The set of equations (2.3), (2.5), (2.6), (3.9), (3.10), (4.2), (4.3), (4.4) and (4.5) with $di_d/dt = di_D/dt = di_F/dt = di_q/dt = di_Q/dt = 0$, $L_d'' = L_q'' = L_c$, and $R_a = 0$ gives a complete description of the steady-state model of the synchronous machine. This set may be solved numerically.

5. THE DYNAMIC MODEL OF THE SYNCHRONOUS MACHINE WITH DIODE BRIDGE

In the dynamic model of the system, the description of the rectifier in chapter 2 will be used (the ripple on the direct current is neglected). Combining the model from this chapter (figure 5) and figure 2 results in an equation for the dc-circuit (for $I_g > 0$)

$$\frac{dI_g}{dt} = \frac{\left\{ \frac{3}{\pi} \sqrt{3} \hat{e} - \left(\frac{3}{\pi} \omega L_c + R_g \right) I_g - U_b \right\}}{(L_g + 2L_c)} \quad (5.1)$$

As in chapter 4, in the phase currents of the synchronous machine only the basic harmonics are taken into account. However, the amplitude, phase, and angular frequency of these basic harmonics may vary now. So using (2.6) and (4.5), it may be seen that I_q and I_d may vary. Hence, these quantities are no longer the constant parts of i_q and i_d . Now, they may be seen as short-term averaged parts of i_q and i_d . These short-term averaged currents are used in the synchronous machine equations.

As may be seen in figure 7, the (averaged) currents through L_{dF} , L_{dD} , and L_{qQ} might well be chosen as state variables:

$$I_{dF} = I_d + I_F ; \quad I_{dD} = I_d + I_D ; \quad I_{qQ} = I_q + I_Q \quad (5.2)$$

Using these expressions and (4.2) with $R_a = 0$ ($L_q'' = L_d''$), after some manipulations the sets of equations (3.9) and (3.10) become

$$e_q = \omega (L_{dF} I_{dF} + L_{dD} I_{dD}) + R_Q (I_{qQ} - I_q) \quad (5.3a)$$

$$e_d = -\omega L_{qQ} I_{qQ} - u_F + R_F (I_{dF} - I_d) + R_D (I_{dD} - I_d) \quad (5.3b)$$

$$\frac{dI_{qQ}}{dt} = -R_Q (I_{qQ} - I_q) / L_{qQ} \quad (5.3c)$$

$$\frac{dI_{dF}}{dt} = [(L_{dD} + L_{FD}) \{u_F - R_F (I_{dF} - I_d)\} - L_{FD} R_D (I_{dD} - I_d)] / (L_{dD} L_{dF} + L_{dD} L_{FD} + L_{FD} L_{dF}) \quad (5.3d)$$

$$\frac{dI_{dD}}{dt} = [L_{FD} \{u_F - R_F (I_{dF} - I_d)\} - (L_{dF} + L_{FD}) R_D (I_{dD} - I_d)] / (L_{dD} L_{dF} + L_{dD} L_{FD} + L_{FD} L_{dF}) \quad (5.3e)$$

The set of equations (2.3), (2.6), (4.4), (4.5), (5.1), and (5.3) gives a description of the dynamic model of the synchronous machine with dc-link. This is a fourth order model with I_g , I_{qQ} , I_{dF} , and I_{dD} as state variables. Unfortunately on the moment of writing this paper, the author had not found a method to give the differential equations in an explicit form, so that the set of equations (2.3), (2.6), (4.4), (4.5), (5.3a), and (5.3b) had to be solved numerically in each integration step. However, when the Newton Raphson iteration method is used, this is not a real problem, because the solution is found in only a few steps.

6. EXAMPLES

In order to give an impression of the value of this model, the transients after a normal switch-on of a 20 kW system have been computed in two different ways. In figure 9 some results of the computation using the model presented here are given. These transients have been computed by means of the simulation package ACSL on a personal computer. The computation time is about 80 s. In figure 10 the results of a detailed simulation on a main-frame computer are presented [2]. In this case the computation time was about 1000 s. Comparing these two figures, it may be seen that the differences, except for the ripple in figure 10, are very small. In order to spare computation time for the detailed simulation, the rotor of the synchronous machine has been assumed to rotate with a constant angular velocity.

7. CONCLUSION

In this paper a rather simple fourth-order dynamic model of a synchronous machine with dc-link is presented. Using this model instead of a more complex model for a detailed simulation results in an enormous reduction of computation time at the expense of information about details, such as harmonics on the phase currents and the ripple on the direct current. However, short-term averaged values of the system variables are simulated correctly.

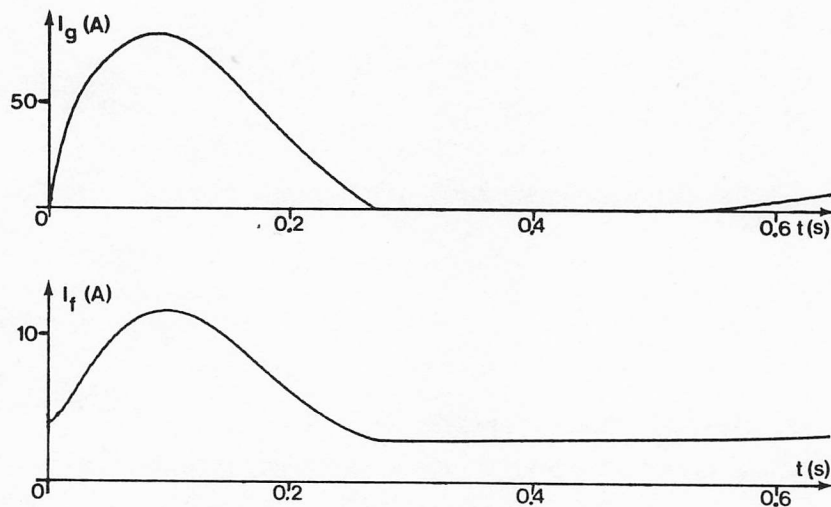


Figure 9 A normal switch-on, computed with the model presented here
(Computation time: 80 s on a personal computer)

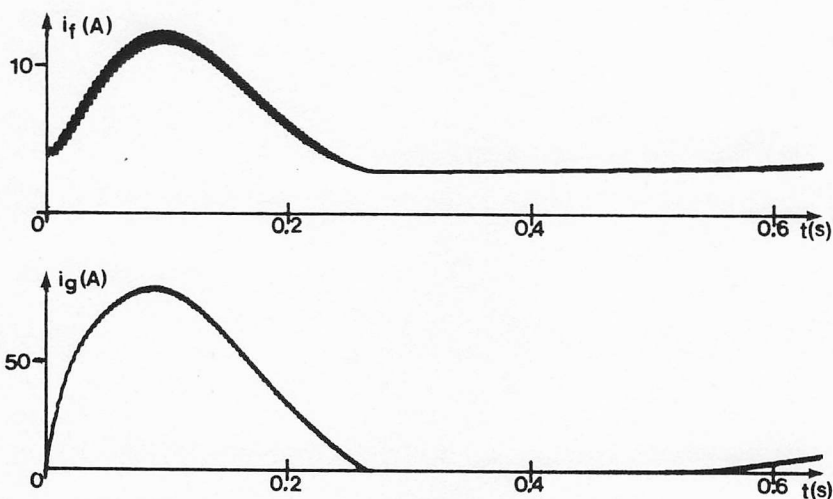


Figure 10 A normal switch-on, detailed simulation
(Computation time: 1000 s on a main-frame computer)

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