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A simple model of a synchronous machine with convertor

## A SIMPIE MODEI OF A SIINCHRONOUS MACHINE: WITTH CONVERTOR

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## 1 INTRODUCTION

The interest in renewable energy has resulted in much research in wind-energy conversion-systems. One of the favourite conversion systems is the series system synchronous machine - rectifier smoothing coil - inverter as depicted in figure 1 , by means of which variable-speed operation of the wind turbine is possible, so that wind energy as well as system components may be utilized in an optimal way.


Figure 1 A wind-energy conversion-system with synchronous machine and dc-1ink

Although the steady-state behaviour of this system is good, its transient behaviour may be problematic, especially in case of a large system. In order to investigate the dynamic behaviour of this system, a detailed simulation of the system is often used. Such a simulation, in which for example the commutation in the rectifier can be recognized, requires a lot of computation time. However, in order to investigate the stability, not only the electrical part, but the whole wind-energy conversion-system, so including the mechanical part, has to be considered. Using a detailed simulation of the synchronous machine with rectifier, the computation time would be too large for normal use.
In an earlier paper, the principles of a less detailed simulation method have been presented for the case of a synchronous machine with a diode bridge rectifier [1]. In this paper, a global dynamic simulation model for a synchronous machine with controllable rectifier (convertor) is derived. Hence, the model presented here may also be used when the synchronous machine works as a motor. In contrast with [1], machine fluxes are used in stead of machine currents as state variables.

The system considered here is given in figure 2. After having described the convertor, some attention will be paid to the coupling of the synchronous machine model with the rectifier model. Next, a suitable set of equations for the synchronous machine will be derived. This set will first be used for a steady-state model of the machineconvertor combination. Using the suppositions from this model, the dynamic model will be derived.


Figure 2 The system considered here

## 2 THE THREE-PHASE BRIDGE CONVERTOR

In the description of the convertor, the circuit shown in figure 3 will be used. The convertor is fed by a three-phase voltage source with internal self-inductance $L_{c}$ and internal voltages $e_{a}, o_{b}$, and $e_{c}$ according to
$\hat{e}_{a}-\hat{e x} \cos (\omega t) ; e_{b}-\hat{e x} \cos \left(\omega t-\frac{2}{3} \pi\right) ; e_{c}-\hat{e c o s}\left(\omega t-\frac{4}{3} \pi\right)$
where $\omega$ is a constant angular frequency and $\hat{e}$ is a constant amplitude. The convertor is loaded by a constant current source $I_{g}$. The thyristors will be considered as ideal switches; resistances in the circuit are neglected.
Each $\pi / 3$ rad a thyristor is triggered. The convertor is controlled by varying the delay angle $a$ : the angle by which the triggering instant is delayed with respect to the starting instant of the conduction of this thyristor in the case all thyristors are continuously triggered, i.e. the thyristors act like diodes. Hence a diode bridge rectifier corresponds with a convertor with $\alpha=0$.


Figure 3 Base circuit for the convertor description

Thanks to the symmetry of the circuit and of the currents and voltages in this circuit, the description of the convertor can be restricted to an interval of $x / 3$ rad. Here, the interval between the triggering instant of thyristor $T_{1}$ and the triggering instant of thyristor $T_{6}$ will be used: $-\pi / 3+\infty<\omega t<a$. This interval is indicated by means of a thick line piece in figure 4. The angle of overlap $\mu$, which will be defined later on, is supposed to be smaller than $\pi / 3 \mathrm{rad}$.


Figure 4 Some quantities as functions of we ( $\alpha=0.3$; $\mu=0.4$ )

Just before the considered interval, the thyristors $T_{4}$ and $T_{5}$ are conducting; at the beginning of this interval thyristor $T_{1}$ will turn on and the current $I_{g}$ starts to transfer from thyristor $T_{5}$ to thyristor $T_{1}$ (the starting instant of the commutation). During this commutation only the thyristors $T_{1}, T_{4}$, and $T_{5}$ are conducting. Hence, using (1) and the initial condition $i_{a}(-\pi / 3+\alpha)-0$, the following relations can be given for the commutation interval considered:
$i_{a} \frac{\sqrt{3} \hat{e}}{2 \omega L_{c}}\left(\cos \alpha-\cos \left(\omega t+\frac{\pi}{3}\right)\right) \quad ; \quad i_{b}=-I_{g}$
$i_{c}=I_{g}-\frac{\sqrt{3} \hat{e}}{2 \omega L_{c}}\left(\cos \alpha-\cos \left(\omega t+\frac{\pi}{3}\right)\right) \quad ; \quad u_{g}-\frac{3}{2} \hat{e} \cos \left(\omega t+\frac{\pi}{3}\right)$
The commutation is finished when the current
through thyristor $T_{5}\left(i_{c}\right)$ becomes zero. The time expressed in angular measure, elapsed from the beginning of the commutation until the end of the commutation is called the angle of overlap $\mu$. In the considered interval the commutation is finished at the instant corresponding to $\omega t-\pi / 3+\alpha+\mu$. From the condition $i_{c}(-\pi / 3+\alpha+\mu)-0$ and (2), it follows:
$\cos \alpha-\cos (\alpha+\mu)=\frac{2 \omega L_{c}{ }_{\mathrm{C}}^{\mathrm{g}}}{}$
After the commutation being finished, only the thyristors $T_{1}$ and $T_{4}$ are conducting. Using figure 3 and the voltage expressions (1), the following expressions can be given (only valid in the second part of the interval considered):
$i_{a}=-i_{b}-I_{g} ; \quad i_{c}-0 ; \quad u_{g}-e_{a}-e_{b}-\sqrt{3 e \cos \left(\omega t+\frac{\pi}{6}\right)}$
The average value of the voltage $u_{g}$ can be found by means of the expressions (2), (4), and (3):
$U_{g 0}-\frac{3}{\pi} \int_{\alpha-\frac{\pi}{3}}^{\alpha} u_{g} d \omega t-\frac{3}{\pi} \sqrt{3} \hat{e} \cos \alpha-\frac{3}{\pi} \omega L_{c} I_{g}$
By means of Fourier analysis and the equations (2), (3), and (4), the fundamental components of the phase currents may be expressed as
$i_{a 1}(\omega t)=i_{a c t} \cos (\omega t)+i_{r e a} \sin (\omega t)$
$i_{b 1}(\omega t)-i_{a c t} \cos \left(\omega t-\frac{2}{3} \pi\right)+i_{\text {rea }} \sin \left(\omega t-\frac{2}{3} \pi\right)$
$i_{c 1}(\omega t)-i_{a c t} \cos \left(\omega t-\frac{4}{3} \pi\right)+i_{\text {rea }} \sin \left(\omega t-\frac{4}{3} \pi\right)$
where the active and the reactive component coeffi cients are given by
$i_{\text {act }}-\frac{\sqrt{3}}{\pi} I_{g}(\cos \alpha+\cos (\alpha+\mu))$
$i_{r e a}=\frac{3 e}{2 \omega L_{c} \pi}(\mu-\sin \mu \cos (2 \alpha+\mu))$
In many practical situations, the ripple on the direct current may be neglected, so that the above description may be used for the steady state. The description may also be used for slow changes in the amplitude or the frequency of the phase voltages and the average value of the direct current.

The dynamic model introduced in this way may be improved by enlarging the inductance in the dccircuit with $2 L_{c}[2]$. This enlargement corresponds to the inductance seen from the dc-side of the convertor when two thyristors are conducting. Using (5), the equivalent circuit given in figure 5 may be composed.


Figure 5 An equivalent circuit for the convertor

3 THE COUPLING OF THE MODEL of the synchronous machine and the model of the convertor

In order to use this conventional circuit of the rectifier for the combination of a synchronous machine with convertor, the standard model of the synchronous machine should be adapted. A simple way to do this will be explained for the steady-state in this chapter. In the next chapters the treatment will be more profound.
First, it is supposed that the subtransient directaxis inductance equals the subtransient quadratureaxis inductance: $\mathrm{L}_{\mathrm{d}}^{\mathrm{d}} \mathrm{L}_{\mathrm{q}}^{\prime \prime}-\mathrm{L}^{n}$.
When the synchronous machine with convertor is in steady-state operation, it can be proven that each armature current may be described by means of a Fourier series which consists of a fundamental component with angular frequency $\omega$ and harmonics with angular frequencies of ( $6 k-1$ ) $\omega$ and $(6 k+1) \omega$, where $k$ is an integer larger than 0 . Seen from the rotor, the fundamental component in the armature currents is a direct current. So, for the fundamental components of the armature currents the synchronous inductances have to be used. Seen from the rotor, the harmonics in the armature currents are currents with angular frequencies of 6 kw . In practice, these are relatively very fast changing currents, so that for the harmonics in the armature currents the subtransient inductance has to be used.

Hence, except for the fundamental component, the armature current always sees the subtransient inductance. This inductance will be splitted off by subtracting it from the synchronous inductances (see figures 6a and 6b). Here arises the so-called internal machine: the original machine minus the subtransient inductances.
Since the harmonics only see the subtransient inductance, the internal machine (figure 6b) is a short-circuit for these harmonics. Hence, the armature voltage of the internal machine is sinusoidal and may be represented by a three-phase voltage source (figure 6c), which is controlled by the excitation current and the fundamental components of the armature phase currents.
Now, the base circuit for the rectifier (figure 3) is found again. Using this circuit, the fundamental components of the stator current can be computed. These components can be used for the computation of the armature voltages of the internal machine. So, in principle, the problem is solved.
After this incomplete explanation, in the next chapters the coupling will be described in more detail.


Figure 6 Splitting off the subtransient inductance

4 THE SYNCHRONOUS MACHINE

### 4.1 The basic set of equations

As in many cases, in this paper the (salient pole) synchronous machine is represented with one damper winding on the direct axis and one damper winding on the quadrature axis. In order to describe this machine, the Park transformation for the phase current according to
$i_{d}=\frac{\sqrt{2}}{\sqrt{3}}\left(i_{a} \cos \gamma+i_{b} \cos \left(\gamma-\frac{2}{3} \pi\right)+i_{c} \cos \left(\gamma-\frac{4}{3} \pi\right)\right\}$
$i_{q}-\frac{\sqrt{2}}{\sqrt{3}}\left(i_{a} \sin \gamma+i_{b} \sin \left(\gamma-\frac{2}{3} \pi\right)+i_{c} \sin \left(\gamma-\frac{4}{3} \pi\right)\right\}$
$i_{0}-\frac{1}{\sqrt{3}}\left(i_{a}+i_{b}+i_{c}\right)$
will be used. The angle $\gamma$ is defined in figure 2. For the phase voltages and flux linkages simular formulas will be used. As may be seen in figure 2 , the homopolar current is zero. For that reason no attention is paid to this component. With the usual suppositions for synchronous machines (see e.g. [3]), the stator voltage equations may be given by
$u_{d}=-R_{a} i_{d}-\frac{d \Psi}{d t}-\omega \Psi_{q} ; \quad u_{q}=-R_{a} i_{q}-\frac{d \Psi}{d t}+\omega \Psi{ }_{d}$
where $\omega$ is the angular speed of the rotor $(\omega=d y / d t)$. The rotor voltage equations are
$u_{f}-R_{f} f_{f}+\frac{d \Phi_{f}}{d t} ; 0-R_{11 d^{i}} 1 d^{+} \frac{d \Psi_{1 d}}{d t} ; 0-R_{11 q^{i}} 1 q+\frac{d \Psi^{1 q}}{d t}$
In these expressions the subscripts $f, 1 d$ en $1 q$ refer to, respectively, the excitation (field) winding, the damper winding on the direct axis, and the damper winding on the quadrature axis.
The flux linkages in the voltage equations are given by
$\Psi_{d}-L_{d} i_{d}+L_{a f d^{i}}+L_{a l d^{i} l d}$
$\Psi_{f}-L_{a f d^{i} d}+L_{f f d^{i} f}+L_{f 1 d^{i} 1 d}$
$\Psi_{1 d}-L_{a 1 d}{ }^{i} d+L_{f 1 d^{i}}{ }_{f}+L_{11 d^{i} 1 d}$
$\Psi_{q}=L_{q} i_{q}+L_{a 1 q}{ }^{i} 1 q$
$\Psi_{1 q}-L_{a 1 q} i_{q}+L_{11 q}{ }^{i}{ }_{1 q}$
The per-unit system will not be used in this paper, so that the base values for voltage, current and time for the stationary circuit are, respectively, 1V, 1A, and 1s. However, in order to get equivalent circuits suitable for simulation, the rotor base values for voltage and for current are chosen different. This is realized by introducing three new currents: one for the excitation winding, one for the direct-axis damper winding, and one for the quadrature-axis damper winding
$i_{F}-\frac{1}{C_{F}} i_{f} \quad ; \quad i_{D}=\frac{1}{C_{D}} . i_{1 d} \quad ; \quad i_{Q}-\frac{1}{C_{Q}} i_{1 q}$
The corresponding fluxes and excitation voltage are $\Psi_{F}=C_{F} \Psi_{f} ; \quad \Psi_{D}-C_{D} \Psi_{1 d} ; \quad \Psi_{Q}-C_{Q}{ }^{\Psi} 1 q ; \quad u_{F}-C_{F} u_{f}$
In order to get simple equations, the following parameters are introduced:
$R_{F}-C_{F}^{2} R_{f} \quad ; \quad L_{F}-C_{F}^{2} L_{f f d} ; L_{a F}-C_{F} L_{a f d}$;
$R_{D}-C_{D}^{2} R_{11 d} ; L_{D}-C_{D}^{2} L_{11 d} ; L_{a D}-C_{D} L_{a 1 d}$;
$L_{D F}-C_{D} C_{F} L_{f l d}$;
$R_{Q}-C_{Q}^{2} R_{11 q} ; L_{Q}-C_{Q}^{2} L_{11 q} ; L_{a Q}-C_{Q} L_{a 1 q}$
Using (12), (13), and (14) the rotor voltage equations (10) become
$u_{F}-R_{F} i_{F}+\frac{d \Psi}{d t} ; \quad 0-R_{D} i_{D}+\frac{d \Psi}{d t} ; 0-R_{Q} i_{Q}+\frac{d \Psi}{d t}$
and the expressions for the fluxes (11) become
$\Psi_{d}-L_{d} i_{d}+L_{a F} i_{F}+L_{a D}{ }^{i} D$
$\Psi_{F}-L_{a F} i_{d}+L_{F} i_{F}+L_{D F} i_{D}$
$\Psi_{D}-L_{a D}{ }^{i} d+L_{D F} i_{F}+L_{D} i_{D}$
$\Psi_{q}-L_{q} i_{q}+L_{a Q^{i}} Q_{i}$
$\Psi_{Q}=L_{a Q_{q}}{ }_{q}+L_{Q \cdot Q}$

## 4. 2 Splitting off the subtransient inductances

When the subtransient inductances have to be

$$
\begin{align*}
& \text { splitted off, the choice } C_{F}=\frac{L_{a f d} L_{11 d}-L_{a 1 d} L_{f 1 d}}{L_{f f d^{\prime}} L_{11 d}-L_{f 1 d}^{2}} ; \\
& C_{D}=\frac{L_{a 1 d} L_{f f d}-L_{a f d} L_{f 1 d}}{L_{f f d^{\prime}} L_{11 d}-L_{f 1 d}^{2}} ; C_{Q}-\frac{L_{a 1 q}}{L_{11 q}} \tag{17}
\end{align*}
$$

may be convenient [4]. Using this choice and the definitions of the new parameters (14), the following relations between these parameters may be derived:
$L_{F}-L_{a F}-L_{D F} ; L_{D}=L_{a D}-L_{D F} ; L_{Q}-L_{a Q}$
Substituting these relations into (16) results into
$\Psi_{d}=L_{d} i_{d}+\quad L_{a F} i_{F}+\quad L_{a D} i_{D}$
${ }_{F}-L_{a F} i_{d}+\left(L_{a F}-L_{D F}\right) i_{F}+\quad L_{D F} i_{D}$
$\Psi_{D}-L_{a D}{ }_{d}+\quad L_{D F} i_{F}+\left(L_{a D}-L_{D F}\right) i_{D}$
$q_{q}-L_{q} i_{q}+L_{a Q} i_{Q}$
$\Psi_{Q}-L_{a Q_{q}^{i}}+L_{a Q_{Q}}^{i}$
After introducing the parameters
$L_{q}^{\prime \prime}-L_{q}-L_{a Q} ; L_{d}^{\prime \prime}-L_{d}-L_{a F}-L_{a D}$
the equivalent circuits in figure 7 may be derived from (9), (15), (19), and (20).


Figure 7 The direct-axis (a) and the quadratureaxis (b) equivalent circuits

As the sub-transient inductance corresponds to the inductance seen in case of a very rapidly changing current, it is easy to see that $L_{d}^{m}$ in figure 7a and $L_{q}^{n}$ in figure $7 b$ are, respectively, the normal direct-axis and the normal quadrature-axis subtransient inductances.
In this paper ${ }_{F}{ }^{\prime}{ }^{W}{ }_{D},{ }_{Q}, I_{d}$, and $I_{q}$ are chosen as state-variables for the description of the synchronous machine. Using (18) and (20), the following differential equations may be derived from (9), (15), and (19)
$\frac{d_{F}}{d t}-u_{F}+R_{F} i_{d}-R_{F D} \frac{L_{D}-L_{D F} L_{D}}{L_{F} L_{D}-L_{D F}^{2}}$
$\frac{d \Psi_{D}}{d t}=R_{D} i_{d}-R_{D} \frac{L_{F} \Psi_{D}-L_{D F} \Psi_{F}}{L_{F} L_{D}-L_{D F}^{2}}$
$\frac{d \Psi_{Q}}{d t}-R_{Q_{q}}^{I_{q}}-\frac{R_{Q}}{L_{a Q}}{ }_{Q}$
$L L_{d}^{\prime \prime} \frac{d i_{d}}{d t}=-R_{a} i_{d}-\frac{d \Psi}{d t}-\frac{d \Psi}{d t}-\omega\left(\Psi_{Q}+L_{q}^{n} i_{q}\right)-u_{d}$
(22a)
$\underset{q}{ } \frac{d q_{q}}{d t}--R_{a_{q}} i_{q}-\frac{d \Psi}{d t}+\omega\left(\Psi_{F}+\Psi_{D}+L_{q}^{\prime \prime} I_{q}\right) \quad-u_{q}$
The relations between the direct-axis and the quadrature-axis voltages and the phase voltages are given by the Park transformation according to
for the voltages with $u_{0}-0$ :
$u_{a}-\frac{\sqrt{2}}{\sqrt{3}}\left(u_{d} \cos \gamma+u_{q} \sin \gamma\right)$
$u_{b}=\frac{\sqrt{2}}{\sqrt{3}}\left(u_{d} \cos \left(\gamma-\frac{2}{3} \pi\right)+u_{q} \sin \left(\gamma-\frac{2}{3} \pi\right)\right\}$
$u_{c}=\frac{\sqrt{2}}{\sqrt{3}}\left(u_{d} \cos \left(\gamma-\frac{4}{3} \pi\right)+u_{q} \sin \left(\gamma-\frac{4}{3} \pi\right)\right)$
Using (8), $\mathrm{i}_{0}-0$, the supposition
$L_{d}^{\prime \prime}-L_{q}^{\prime \prime}=L^{\prime \prime}$
and the internal voltages
$e_{d}-u_{d}+R_{a} i_{d}+L \frac{d i_{d}}{d t}+\omega L^{\prime \prime} i_{q}$
$e_{q}-u_{q}+R_{a_{q}} i_{q}+L{ }^{\prime \prime} \frac{d i_{q}}{d t}-\omega L^{\prime \prime} i_{d}$
the phase voltage according to (23) become
$u_{a}-\frac{\sqrt{2}}{\sqrt{3}}\left(e_{d} \cos \gamma+e_{q} \sin \gamma\right) \quad-R_{a} i_{a}-L \frac{d i}{d t} a^{d} \quad$ (26a)
$u_{b}-\frac{\sqrt{2}}{\sqrt{3}}\left(e_{d} \cos \left(\gamma-\frac{2}{3} \pi\right)+e_{q} \sin \left(\gamma-\frac{2}{3} \pi\right)\right)-R_{a} i_{b}-L^{\prime \prime} \frac{d i_{b}}{d t}$
$u_{c}=\frac{\sqrt{2}}{\sqrt{3}}\left(e_{d} \cos \left(\gamma-\frac{4}{3} \pi\right)+e_{q} \sin \left(\gamma-\frac{4}{3} \pi\right)\right)-R_{a} i_{c}-L \frac{d i}{d t}$
After introducing, by means of the Park transformation, the internal voltages
$e_{a}-\frac{\sqrt{2}}{\sqrt{3}}\left(e_{d} \cos \gamma+e_{q} \sin \gamma\right)$
$e_{b}=\frac{\sqrt{2}}{\sqrt{3}}\left(e_{d} \cos \left(\gamma-\frac{2}{3} \pi\right)+e_{q} \sin \left(\gamma-\frac{2}{3} \pi\right)\right\}$
$e_{c}=\frac{\sqrt{2}}{\sqrt{3}}\left(e_{d} \cos \left(\gamma-\frac{4}{3} \pi\right)+e_{q} \sin \left(\gamma-\frac{4}{3} \pi\right)\right\}$
these expressions may be written as
$u_{a}=e_{a}-R_{a} i_{a}-L " \frac{d i}{d t}{ }^{\prime}$
$u_{b}-e_{b}-R_{a} i_{b}-L \frac{d i_{b}}{d t}$
$u_{c}-e_{c}-R_{a} i_{c}-\frac{d i}{d i} \frac{c_{c}}{d t}$
From this set of equations, the set (25), and figure 7, the set of circuits given in figure 8 may be derived with (24). The coupling between the circuits given in figure 8 is given by the sets of equations (8) and (2?).

## 4,3 The equations for the simple model

In order to get a simple set of equations, the voltages $u_{d}$ and $u_{q}$ are eliminated from the equations (25) and (22) with (24):
$e_{d}=-\frac{d \Psi_{F}}{d t}-\frac{d \Psi_{D}}{d t}-\omega \Psi_{Q}$
$e_{q}=-\frac{d \Psi_{Q}}{d t}+\omega\left(\Psi_{F}+\Psi_{D}\right)$
The total set of synchronous machine equations exists of (8), (21), (27), (28), and (29).


Figure 8 Equivalent circuit of the synchronous machine with $L^{\prime \prime}=L_{d}^{\prime \prime}=L_{q}^{n}$

5 THE STEADY-STATE MODEL OF THE SYNCHRONOUS MACHINE WITH CONVERTOR

In order to combine the models in the chapters 2 and 4 according to the method explained in chapter 3 , the equivalent circuit of the synchronous machine should correspond with figure 3 . As is shown in chapter 4 (figure 8), this may easily be achieved when $L_{d}^{\prime \prime}-L_{q}^{\prime \prime}-L^{\prime \prime}$. In most other cases ( $L_{d^{\prime \prime}}{ }^{\left(L_{q}^{\prime \prime}\right)}$ an approximation can be used [5]. Besides, the armature resistance has to be neglected ( $R_{a} m 0$ ), which is a normal supposition for synchronous machine description.
As has been mentioned before, the direct-axis and the quadrature-axis components of the armature currents consist of a constant part ( $\mathrm{I}_{\mathrm{d}}$ and $\mathrm{I}_{\mathrm{q}}$ ) and a fast changing part which may be represented by a Fourier series with terms with an angular frequency which is a multiple of $6 \omega$. For these fast changing current components the network parts to the right of the dashed lines in figure 8 (compare with figure 7) resemble short circuits. This resemblance is supposed to be exact, so that the dashed lines can be interpreted as short circuits for these components. As a result, $e_{d}$ and $e_{q}$ are constant, so that the voltages $e_{a}, e_{b}$, and $e_{c}$ in the lower part of figure 8 are sinusoidal. Hence, this part corresponds to figure 3. When the position angle $\gamma$ is chosen as $\gamma=\omega t+\pi / 2$, these voltages may be expressed
as:
$e_{a}=\hat{e c o s}(\omega t-\epsilon)$
$e_{b}=\hat{e c o s}\left(\omega t-\epsilon-\frac{2}{3} \pi\right)$
$e_{c}=\hat{e} \cos \left(\omega t-\epsilon-\frac{4}{3} \pi\right)$ (30c)
where $\hat{e}=\frac{\sqrt{2}}{\sqrt{3}} \sqrt{e_{d}^{2}+e_{q}^{2}}$
The only difference between these expressions and (1) is the phase shift $\epsilon$, which represents the load angle of the internal machine of figure 6 .

In order to compute the steady-state of the synchronous machine with convertor, the constant parts of $i_{d}$ and $i_{q}, I_{d}$ and $I_{q}$, have to be known. These constant parts correspond to the basic harmonics of the armature phase currents, so that $I_{d}$ and $I_{q}$ may be found by substituting (6) into (8) with $\gamma-\omega t+\pi / 2$ and $\omega t$ in (8) replaced by $\omega t-\epsilon$ :
$I_{d}=-\frac{\sqrt{3}}{\sqrt{2}}\left(i_{a c t} \sin \epsilon+i_{r e a} \cos \epsilon\right)$
$I_{q}=\frac{\sqrt{3}}{\sqrt{2}}\left(i_{a c t} \cos \epsilon-i_{r e a} \sin \epsilon\right)$
The set of equations (3), (5), (7), (21), (29), (31), and (32) with $d \Psi_{F} / d t=d \Psi_{D} / d t-d \Psi_{Q} / d t-0, R_{a}-0$, and, $L_{d}^{n}=L_{q}^{\prime \prime}=L_{c}$ gives a complete description of the steady-state model of the synchronous machine with convertor. This set may be solved numerically.

6 THE DYNAMIC MODEL OF THE SYNCHRONOUS MACHINE WITH CONVERTOR

In the dynamic model of the system, the description of the rectifier in chapter 2 will be used (the ripple on the direct current is neglected). Combining the model from this chapter (figure 5) and figure 2 results in an equation for the dc-circuit (for $I_{g}>0$ )
$\frac{d I_{g}}{d t}=\left(\frac{3}{\pi} \sqrt{3 \hat{e} \cos \alpha}-\left(\frac{3}{\pi} \omega L_{c}+R_{g}\right) I_{g}-U_{b}\right) /\left(L_{g}+2 L_{c}\right)$
As in chapter 5 , in the phase currents of the synchronous machine only the basic harmonics are taken into account. However, the amplitude, phase and angular frequency of these basic harmonics may vary now. So using (6) and (32), it may be seen that $I_{d}$ and $I_{q}$ may vary. Hence, these quantities are no longer the constant parts of $i_{q}$ and $i_{d}$. Now, they may be seen as short-term averaged parts of $i_{9}$ and $i_{d}$. These short-term averaged currents are used in the synchronous machine equations.
Hence, the set of equations (3), (7), (21), (29), (31), (32), and (33) gives a description of the dynamic model of the synchronous machine with dclink. This is a fourth order model with $I_{g}, \Psi_{F}, \Psi_{D}$, and $\Psi_{Q}$ as state variables. Since there has not be found a method to give the differential equations in an explicit form until now, the set of equations (3), (7), (29), (31), and (32) has to be solved
numerically in each integration step. However, when the Newton-Raphson iteration method is used, this is not a real problem, because the solution is found in only a few steps.

## CONCLUDING REMARKS

In this paper a rather simple fourth-order dynamic model of a synchronous machine with de-link is presented. For the case where the convertor consists of diodes, the results obtained by means of this simple model have been compared with the results of a more complex, detailed model [1]. It appeared, that using the simple model presented here instead of the more complex model results in an enormous reduction of computation time at the expense of information about details, such as harmonics on the phase currents and the ripple on the direct current. However, short-term averaged values of the system variables are simulated correctly.

At the moment of writing this paper, research attention was aimed at the experimental validation of the model presented here.

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