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SYNCHRONOUS MACHINE EQUIVALENT CIRCUITS FOR A SYNCHRONOUS MACHINE WITH CONVERTOR

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ABSTRACT

When a synchronous machine which is connected with a convertor is modelled by means of equivalent circuits, the explicit presence of the subtransient inductances in these circuits may be very useful.
Based on the basic set of equations of the synchronous machine, some equivalent circuits are derived, with increasing degree of complexity. The technique used to arrive at these circuits is the well-known transformation technique of referring rotor quantities to the stator. With an appropriate choice of the transformation constants, the subtransient inductance is explicitly present in the equivalent circuits.
This leads to a new three-winding and a new four-winding direct-axis equivalent circuit with explicit subtransient inductance. These equivalent circuit models are derived and discussed.

INTRODUCTION

For synchronous machines, many kinds of equivalent circuits exist. These circuits differ from each other in the way in which rotor quantities are referred to the stator. When the synchronous machine is connected with a power electronic convertor, not all these circuits are equally well suited to describe the machine. As will be explained later on, it is very useful to have equivalent ciruits in which the subtransient (or even subsubtransient) inductances are explicitly present for those cases. Such equivalent circuits have been presented in earlier papers (e.g. [1] or [2]).
These earlier developed circuits, however, have two minor disadvantages. The first is that the excitation winding quantities are referred to the stator, which is not desirable in many cases; the second is that the (physical) meaning of the elements in the circuit is not clear. Besides these disadvantages, the circuits may not easily be extended with more damper circuits, which may sometimes be necessary to model a machine more accurately.
After an explanation of the usefulness of the explicit presence of the subtransient inductances in the next section, and a description of the basic set of equations of the synchronous machine in the subsequent section, some equivalent cicuits will be derived in increasing degree of difficulty. First, the well-known cicuits for the quadrature axis with one damper winding are derived. Next, a new three-winding and a new four-winding equivalent circuit with explicit subtransient inductance for the direct axis of the synchronous machine are given. Using these new circuits, the disadvantages mentioned earlier may be overcome.

THE SYNCHRONOUS MACHINE WITH CONVERTOR

Steady-state operation
Before turning to the topic of equivalent circuits, the need for an explicit subtransient inductance will be discussed in further detail.
In this paper, the (two-pole) synchronous machine is supposed to be connected to a power electronic convertor (see figure 1) which produces a symmetrical three-phase system of currents during steady-state operation.

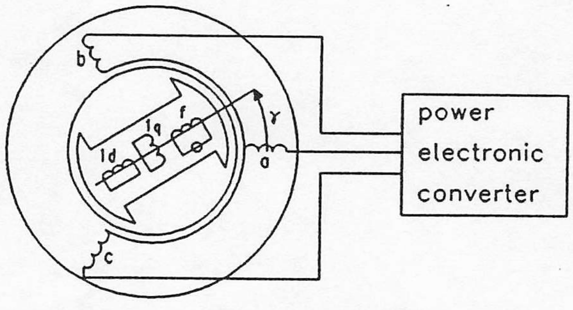

Figure 1 The synchronous machine with convertor

These currents may be expressed as Fourier series in which all even harmonics are zero thanks to the property $i(\omega t-\pi)=i(\omega t)$, where ω represents the angular speed of the rotor. Moreover, as the star connection terminal of the machine is not used, the armature phase currents do not contain harmonics with an

angular frequency which is a multiple of 3ω. Hence, the Fourier series consists of a fundamental component with angular frequency ω and harmonics with angular frequencies of $(6k-1)\omega$ and $(6k+1)\omega$, where k is an integer larger than 0.
Since the rotor "sees" the fundamental components of the phase currents as direct currents, they don't induce currents in the rotor circuits. Hence, the impedance of the stator for the fundamental components is determined by the synchronous inductances. Seen from the rotor, the harmonics in the armature currents are transformed into currents with angular frequencies of $6k\omega$. So, these harmonic currents induce currents in the rotor circuits. Because these currents have a relatively high angular fequency, the harmonics in the armature currents see the subtransient inductance as stator inductance.
In order to demonstrate the profit of making the subtransient inductances explicitly visible in the equivalent circuits, it is - for this explanation only - supposed that the quadrature-axis and the direct-axis synchronous inductances are equal ($L_q=L_d=L_s$) and that the quadrature-axis and the direct-axis subtransient inductances are equal ($L_q''=L_d''=L''$). For the explanation figure 2 will be used. In this figure, a rectifier serves, as an example, as a power electronic convertor.

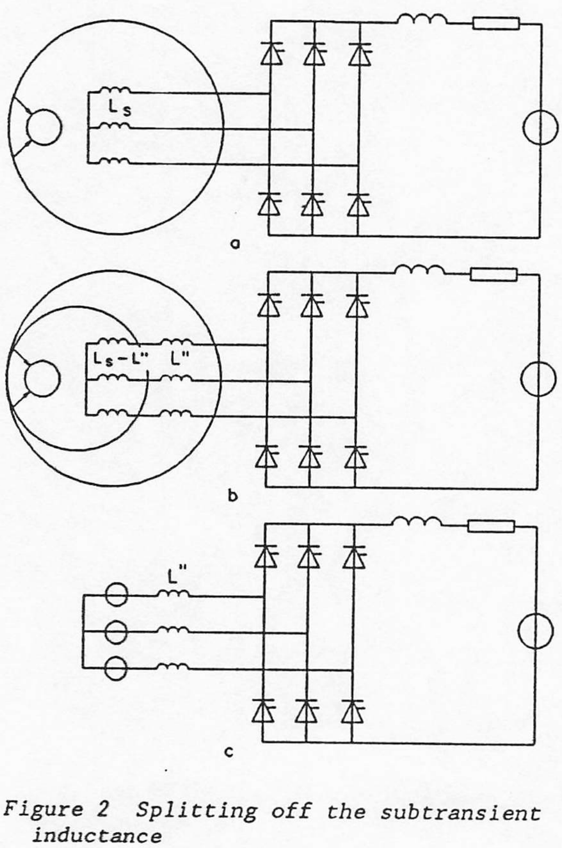

Figure 2 Splitting off the subtransient inductance

In figure 2a the original synchronous machine with synchronous inductance L_s is shown. In figure 2b the subtransient inductance L" is splitted off by subtracting it from the synchronous inductance. The inner circle of figure 2b represents the so-called internal machine: the original machine minus the subtransient inductances.
Since the phase-current harmonics only see the subtransient inductance, the internal machine (figure 2b) is a short circuit for these harmonics. Hence, the armature voltages of the internal machine are sinusoidal and may be represented by a three-phase voltage source (figure 2c), which is controlled by the excitation current and the fundamental components of the armature phase currents. So, the voltages of this source do not depend on the higher harmonic currents.
Splitting off the subtransient inductances effectively separates the fast and the slow modes of the machine, thus offering a possibility to simulate machines in this configuration without considering the switching actions in the power electronic convertor in detail. On the other hand, the three-phase voltage source and the subtransient inductances determine the behaviour of the electronic switches in the convertor.
The assumption for the previous explanation $L_q=L_d$ is not essential. However, when the assumption $L_q''=L_d''$ is not met, the power electronic convertor does not see a constant subtransient inductance any more (for a more comprehensive explanation, see [1]).

Dynamic operation
During dynamic operation, the changes of the fundamental components of the armature phase currents are often slow compared with the frequencies of the harmonics. Hence, since the fundamental components of the armature currents are changing, relatively slow transient currents will occur in the damper windings and in the excitation winding on the rotor. These currents have to be taken into account when determining the voltages of the three-phase voltage source in figure 2c. For this purpose, an equivalent circuit for the synchronous machine with splitted off subtransient inductances may be very useful.
In the considered cases, the three-phase voltage source, with relatively slowly changing amplitude and frequency, and the subtransient inductances still determine the behaviour of the electronic switches in the convertor. Hence, splitting off the subtransient inductances may still be advantageous for investigating the convertor behaviour.
Based on the previous considerations, a simple model of a synchronous machine has been derived [2].

THE BASIC SET OF EQUATIONS

After the discussion of the usefulness of explicit subtransient inductances, attention is focused on the derivation of the equivalent circuits themselves.
In first instance, the (salient pole) synchronous machine is represented with one damper winding on the direct axis and one damper winding on the quadrature axis. In order to describe the machine, the Park transformation according to

$$i_d = \frac{\sqrt{2}}{\sqrt{3}}\{i_a \cos\gamma + i_b \cos(\gamma - \frac{2}{3}\pi) + i_c \cos(\gamma - \frac{4}{3}\pi)\} \quad (1a)$$

$$i_q = \frac{\sqrt{2}}{\sqrt{3}}\{i_a \sin\gamma + i_b \sin(\gamma - \frac{2}{3}\pi) + i_c \sin(\gamma - \frac{4}{3}\pi)\} \quad (1b)$$

$$i_0 = \frac{1}{\sqrt{3}}\{i_a + i_b + i_c\} \quad (1c)$$

will be used. The angle γ is defined in figure 1. For the phase voltages and flux linkages similar formulas will be used. As may be seen in figure 1, the homopolar current is zero. For that reason no attention is paid to this component.
With the usual suppositions for synchronous machines (see e.g. [3]), the stator voltage equations are given by (ω=dγ/dt)

$$u_q = - R_a i_q - \frac{d\psi_q}{dt} + \omega\psi_d \quad (2a)$$

$$u_d = - R_a i_d - \frac{d\psi_d}{dt} - \omega\psi_q \quad (2b)$$

The rotor voltage equations are

$$0 = R_{11q} i_{1q} + \frac{d\psi_{1q}}{dt} \quad (3a)$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \quad (3b)$$

$$0 = R_{11d} i_{1d} + \frac{d\psi_{1d}}{dt} \quad (3c)$$

In these expressions, the subscripts 1q, f, and 1d refer to the damper winding on the quadratue axis, the excitation (field) winding, and the damper winding on the direct axis respectively.
The flux linkages in the voltage equations are given by

$$\psi_q = L_q i_q + L_{a1q} i_{1q} \quad (4a)$$

$$\psi_{1q} = L_{a1q} i_q + L_{11q} i_{1q} \quad (4b)$$

$$\psi_d = L_d i_d + L_{afd} i_f + L_{a1d} i_{1d} \quad (5a)$$

$$\psi_f = L_{afd} i_d + L_f i_f + L_{f1d} i_{1d} \quad (5b)$$

$$\psi_{1d} = L_{a1d} i_d + L_{f1d} i_f + L_{11d} i_{1d} \quad (5c)$$

It should be noted that the per-unit system will not be used in this paper.

THE QUADRATURE AXIS WITH ONE DAMPER WINDING

From the equations (2a), (3a), and (4), the equivalent circuit of the quadrature axis given in figure 3 may be derived. In this circuit, the rotor circuit is not directly accessible. Hence, the real values of the rotor quantities are not of interest (from a system point of view) and it makes sense to refer them to the stator. Besides, it is not possible to determine all parameters in figure 3 from terminal measurements.

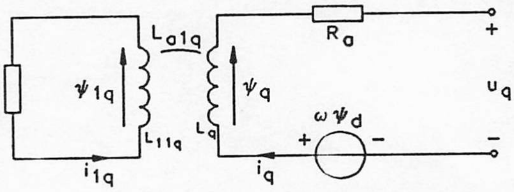

Figure 3 The basic equivalent circuit for the quadrature axis

The rotor quantities are referred to the stator by introducing a new set of rotor quantites

$$\psi_{1Q} = C_{1Q}\,\psi_{1q} \;;\; i_{1Q} = \frac{1}{C_{1Q}}\, i_{1q} \quad (6)$$

where C_{1Q} is a factor which may freely be chosen.
Using (6), the equations (2a), (3a), and (4) may be written as

$$u_q = - R_a i_q - \frac{d\psi_q}{dt} + \omega\psi_d \quad (7a)$$

$$0 = C_{1Q}^2 R_{11q} i_{1Q} + \frac{d\psi_{1Q}}{dt} \quad (7b)$$

$$\psi_q = L_q i_q + C_{1Q} L_{a1q} i_{1Q} \quad (8a)$$

$$\psi_{1Q} = C_{1Q} L_{a1q} i_q + C_{1Q}^2 L_{11q} i_{1Q} \quad (8b)$$

In many cases the quadrature-axis synchronous inductance L_q is divided into a main inductance L_{mq} and a leakage inductance L_σ. In order to find the commonly used transformer circuit for the quadrature axis, the factor C_{1Q} is chosen in such a way that the mutual inductance $C_{1Q}L_{a1q}$ equals the main inductance L_{mq}:

$$C_{1Q} = \frac{L_{mq}}{L_{a1q}} \quad (9)$$

In this way, (8) becomes:

$$\psi_q = L_{a\sigma} i_q + L_{mq}(i_q + i_{1Q}) \quad (10a)$$

$$\psi_{1Q} = (C_{1Q}^2 L_{11q} - L_{mq}) i_q + L_{mq}(i_q + i_{1Q}) \quad (10b)$$

which results into the equivalent circuit given in figure 4. In this case C_{1Q} corresponds with ratio of the number of stator winding turns to the number of rotor winding turns. It should be noted that it is still not possible to determine all parameters in figure 4 by measurement on the stator terminals only.
In order to make the subtransient inductance explicitly visible, C_{1Q} is defined according to

$$C_{1Q} = \frac{L_{a1q}}{L_{11q}} \quad (11)$$

Using (11), (8) may be written as

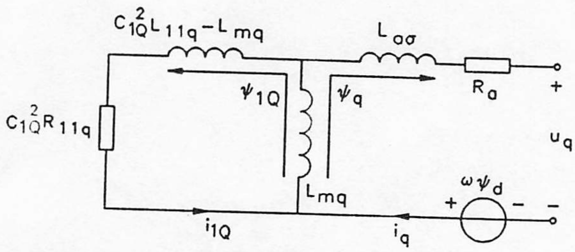

Figure 4 The commonly used equivalent circuit for the quadrature axis

$$\psi_q = (L_q - \frac{L_{a1q}^2}{L_{11q}})i_q + \frac{L_{a1q}^2}{L_{11q}}(i_q + i_{1Q}) \qquad (12a)$$

$$\psi_{1Q} = \frac{L_{a1q}^2}{L_{11q}}(i_q + i_{1Q}) \qquad (12b)$$

After introducing the quantities

$$L_q'' = L_q - \frac{L_{a1q}^2}{L_{11q}} \; ; \; L_{1Q} = \frac{L_{a1q}^2}{L_{11q}} \; ; \; R_{1Q} = \frac{L_{a1q}^2}{L_{11q}^2} R_{11q} \qquad (13)$$

the quadrature-axis equations (7) and (12) become

$$u_q = - R_a i_q - \frac{d\psi_q}{dt} + \omega\psi_d \qquad (14a)$$

$$0 = R_{1Q} i_{1Q} + \frac{d\psi_{1Q}}{dt} \qquad (14b)$$

$$\psi_q = L_q'' i_q + L_{1Q}(i_q + i_{1Q}) \qquad (15a)$$

$$\psi_{1Q} = L_{1Q}(i_q + i_{1Q}) \qquad (15b)$$

By means of this set of equations, the equivalent circuit shown in figure 5 may be constructed.

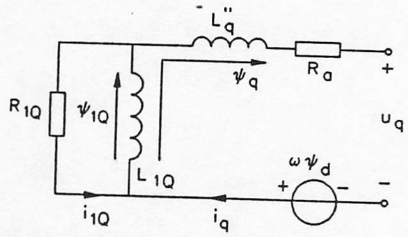

Figure 5 An equivalent circuit for the quadrature axis with the subtransient inductance explicitly present

It is easily verified that very fast changing currents i_q "see" L_q" only. So, L_q" indeed represents the quadrature-axis subtransient inductance .

As contrasted with the parameters in the figures 3 and 4, those in figure 5 may be determined by measurements on the stator terminals only:

R_a: dc resistance;

L_q": inductance seen in case of very fast changing current (the flux ψ_{1Q} may be considered constant);

L_{1Q}: synchronous (dc) inductance L_q minus subtransient inductance L_q" ((13));

R_{1Q}: follows from L_{1Q} and the time constant of the circuit in case of an open stator circuit.

In order to determine these parameters, a practical measuring method has been developed [4].

THE DIRECT AXIS WITH ONE DAMPER WINDING

From the equations (2b), (3b), (3c), and (5) the equivalent circuit in figure 6 may be derived.

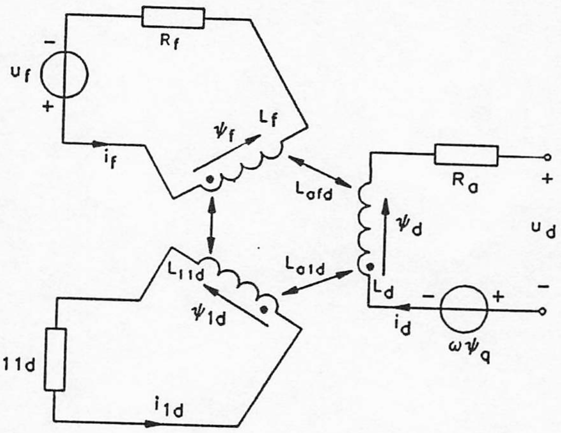

Figure 6 The basic equivalent circuit for the direct axis

As distinct from the quadrature axis, the direct axis has an excitation winding, which is accessible. For that reason, it makes less sense to refer the quantities corresponding with the excitation winding to the stator. However, in an earlier paper, an equivalent circuit with explicit subtransient inductance has been derived in which the excitation winding is also referred to the stator [1,2].

In this paper, only the damper winding quantities are referred to the stator. This is realized by introducing a new set of damper winding quantities

$$\psi_{1D} = C_{1D}\,\psi_{1d} \; ; \; i_{1D} = \frac{1}{C_{1D}}\, i_{1d} \qquad (16)$$

where C_{1D} is chosen according to

$$C_{1D} = \frac{L_{a1d}}{L_{11d}} \qquad (17)$$

in order to find an equivalent circuit with an explicit subtransient inductance. Besides, the parameters

$$L_{1D} = C_{1D}^2\, L_{11d} = \frac{L_{a1d}^2}{L_{11d}} \; ; \; K_{f1D} = \frac{L_{f1d}}{L_{a1d}} \; ;$$

$$L_f' = L_f - K_{f1D}^2 L_{1D} = L_f - \frac{L_{f1d}^2}{L_{11d}} \; ; \; C_F = \frac{L_{afd} - K_{f1D}L_{1D}}{L_f'} \; ;$$

$$L_d'' = L_d - L_{1D} - C_F^2 L_f' \; ; \; R_{1D} = C_{1D}^2 R_{11d} \; ; \qquad (18)$$

are introduced. Now, the direct axis equations (2b), (3b), (3c), and (5) may be written as

$$u_d = -R_a i_d - \frac{d\psi_d}{dt} - \omega\psi_q \qquad (19a)$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \qquad (19b)$$

$$0 = R_{1D} i_{1D} + \frac{d\psi_{1D}}{dt} \qquad (19c)$$

$$\psi_d = L_d'' i_d + C_F L_f'(C_F i_d + i_f) + L_{1D}(i_d + K_{f1D} i_f + i_{1D}) \qquad (20a)$$

$$\psi_f = L_f'(C_F i_d + i_f) + K_{f1D} L_{1D}(i_d + K_{f1D} i_f + i_{1D}) \qquad (20b)$$

$$\psi_{1D} = L_{1D}(i_d + K_{f1D} i_f + i_{1D}) \qquad (20c)$$

These equations are used for the construction of the circuit in figure 7. It should be noted that the transformers indicated with $1:C_F$ and with $K_{f1D}:1$ in this figure are ideally coupled (without leakage).

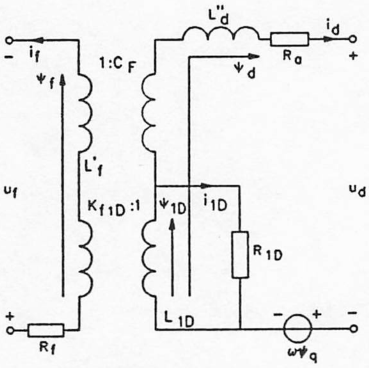

Figure 7 An equivalent circuit for the direct axis with the subtransient inductance explicitly present

The parameters in figure 7 may be determined by measurements on the terminals only [5]:

- R_a: dc stator resistance;
- R_f: dc excitation winding resistance;
- L_d'': stator inductance seen in case of very fast changing stator current with closed excitation winding (the fluxes ψ_{1D} and ψ_f may be considered constant);
- C_F: current ratio $-i_f/i_d$ in case of very fast changing stator current with closed excitation winding (the fluxes ψ_{1D} and ψ_f may be considered constant);
- L_f': excitation winding inductance seen in case of very fast changing excitation current with open stator circuit (the flux ψ_{1D} may be considered constant);
- L_{1D}: follows from the synchronous (dc) inductance L_d by using the expression for L_d'' ((18));
- K_{f1D}: follows from the synchronous (dc) mutual inductance L_{afd} between stator and excitation winding by using the expression for C_F ((18));
- R_{1D}: follows from L_{1D} and the time constant of the circuit in case of an open stator and an open excitation circuit.

THE DIRECT AXIS WITH TWO DAMPER WINDINGS

In many cases modelling the direct axis with only one damper winding is not sufficient [5]. This problem may often be solved by extending the equivalent circuit with an extra damper winding. This means that a damper winding voltage equation should be added to the set of voltage equations (2b), (3b), and (3c):

$$0 = R_{22d} i_{2d} + \frac{d\psi_{2d}}{dt} \qquad (21)$$

and that the flux relations (5) should be extended:

$$\psi_d = L_d i_d + L_{afd} i_f + L_{a1d} i_{1d} + L_{a2d} i_{2d} \qquad (22a)$$

$$\psi_f = L_{afd} i_d + L_f i_f + L_{f1d} i_{1d} + L_{f2d} i_{2d} \qquad (22b)$$

$$\psi_{1d} = L_{a1d} i_d + L_{f1d} i_f + L_{11d} i_{1d} + L_{12d} i_{2d} \qquad (22c)$$

$$\psi_{2d} = L_{a2d} i_d + L_{f2d} i_f + L_{12d} i_{1d} + L_{22d} i_{2d} \qquad (22d)$$

In order to obtain a practical equivalent circuit with explicit subtransient inductance, the damper currents i_{1d} and i_{2d} are transformed into the currents i_{1D} and i_{2D} by using

$$i_{1d} = C_{11} i_{1D} + C_{12} i_{2D} \qquad (23a)$$

$$i_{2d} = C_{21} i_{1D} + C_{22} i_{2D} \qquad (23b)$$

and the damper fluxes ψ_{1d} and ψ_{2d} are transformed into the fluxes ψ_{1D} and ψ_{2D} by using

$$\psi_{1D} = C_{11}\psi_{1d} + C_{21}\psi_{2d} \qquad (24a)$$

$$\psi_{2D} = C_{12}\psi_{1d} + C_{22}\psi_{2d} \qquad (24b)$$

As has been proven in [6], the coefficients C_{11}, C_{12}, C_{21}, and C_{22} may be chosen in such a way, that the voltage equations may be written in the form

$$u_d = -R_a i_d - \frac{d\psi_d}{dt} - \omega\psi_q \qquad (25a)$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \qquad (25b)$$

$$0 = R_{1D} i_{1D} + \frac{d\psi_{1D}}{dt} \qquad (25c)$$

$$0 = R_{2D} i_{2D} + \frac{d\psi_{2D}}{dt} \qquad (25d)$$

and the flux expressions in the form

$$\psi_d = L_d'' i_d + C_F L_f'(C_F i_d + i_f) + L_{2D}'(i_d + K_{f2D}' i_f + i_{2D}) + L_{1D}(i_d + K_{f1D} i_f + i_{1D} + i_{2D}) \qquad (26a)$$

$$\psi_f = L_f'(C_F i_d + i_f) + K_{f2D}' L_{2D}'(i_d + K_{f2D}' i_f + i_{2D}) + K_{f1D} L_{1D}(i_d + K_{f1D} i_f + i_{1D} + i_{2D}) \qquad (26b)$$

$$\psi_{1D} = L_{1D}(i_d + K_{f1D} i_f + i_{1D} + i_{2D}) \qquad (26c)$$

$$\psi_{2D} = L_{2D}'(i_d + K_{f2D}' i_f + i_{2D}) + L_{1D}(i_d + K_{f1D} i_f + i_{1D} + i_{2D}) \qquad (26d)$$

These equations are used for the construction of the circuit in figure 8, where the transformers indicated with $1:C_F$, with $K_{f2D}':1$, and with $K_{f1D}:1$ are ideally coupled (without leakage).

In the equations (25) and (26) a number of new parameters are used. These parameters

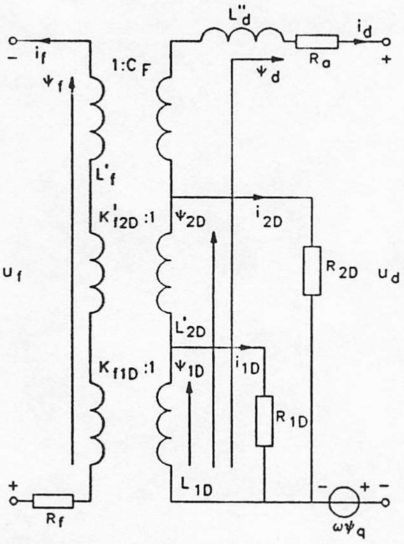

Figure 8 An equivalent circuit for the direct axis with two damper windings and explicit subtransient inductance

may be expressed as functions of the original parameters. However, these functions are rather extensive and are not really important, because the new set of parameters may directly be determined from measurements on the terminals by means of modern parameter estimation techniques [6].
This is in contrast with the original set of parameters, which can not be determined from measurements on the terminals only.
From figure 8, expressions for the (original) dc (synchronous) self and mutual inductances as functions of the new parameters may easily be found:

$$L_d = L_{1D} + L'_{2D} + C_F^2 L'_f + L''_d \qquad (27a)$$

$$L_{afd} = K_{f1D} L_{1D} + K'_{f2D} L'_{2D} + C_F L'_f \qquad (27b)$$

$$L_f = K_{f1D}^2 L_{1D} + K'^2_{f2D} L'_{2D} + L'_f \qquad (27c)$$

It should be noted that the choice for the coefficients C_{11}, C_{12}, C_{21}, and C_{22} used here is rather arbitrary. Other choices may also result in useful equivalent circuits.
Comparison of the figures 7 and 8 leads to the conclusion that the circuit derived here may easily be extended with more damper windings.

CONCLUSION

In this paper a new set of equivalent circuits for the synchronous machine has been presented. These equivalent circuits, which have been based on widely accepted assumptions, have the property that the subtransient inductances are explicitly present. Hence, they are well fitted for cases in which the synchronous machine is used in combination with a power electronic convertor.
The circuits have been derived for the quadrature axis with one damper winding and for the direct axis with one or two damper windings. However, the derivation method may also be used for creating circuits with more damper windings.
Furthermore, the parameters in these equivalent circuits may be determined from terminal measurements only.

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