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A STATE SPACE MODEL OF A SYNCHRONOUS MACHINE WITH CONVERTOR

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ABSTRACT

In order to investigate the dynamic behaviour of the synchronous machine with convertor, a detailed simulation is used in most cases, which requires a lot of computation time. Here, a more superficial simulation method is presented.
In this case the subtransient inductances are (imaginarily) splitted off from the machine. The harmonics on the stator voltages of the remaining (imaginary) machine are neglected, so that these voltages may be supposed to be sinusoidal and may be seen as the voltage sources for the convertor with the subtransient inductances as commutation inductances. Furthermore, the ripple on the direct current is neglected, so that the classic model of a three-phase bridge convertor (including a global modeling of commutation) may be used. On the ac side of the bridge only the basic harmonics of the currents are considered. Using these assumptions, a set of equations for the synchronous machine with convertor is derived. This set is written in the form of state equations with four state variables for the electrical part.

1 INTRODUCTION

The interest in renewable energy has resulted in much research in wind-energy conversion systems. One of the favourite conversion systems is the series system synchronous machine - rectifier - smoothing coil - inverter as depicted in figure 1, by means of which variable-speed operation of the wind turbine is possible, so that wind energy as well as system components may be utilized in an optimal way.

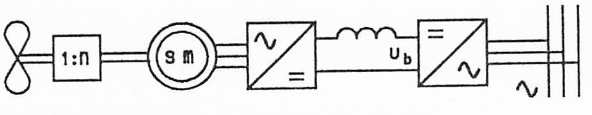

Figure 1 The synchronous machine with dc link in a wind-energy conversion system

Although the steady-state behaviour of this system is good, its transient behaviour may be problematic, especially in case of a large system. In order to investigate the dynamic behaviour of this system, a detailed simulation of the system is often used. Such a simulation, in which for example the commutation in the rectifier can be recognized, requires a lot of computation time. However, in order to investigate the stability, not only the electrical part, but the whole wind-energy conversion-system, so including the mechanical part, has to be considered. Using a detailed simulation of the synchronous machine with convertor, the computation time would be too large for normal use.
In earlier papers, the principles of a less detailed simulation method have been presented [1,2]. However, in the model given in these papers, the state space equations were not given in an explicit form: a numerical solution of the set of equations was necessary in each integration step. In this paper, an explicit form of the state equations is given.
The system considered here is given in figure 2. After having described the convertor and the synchronous machine separately, attention will be paid to the coupling of the synchronous machine model with the convertor model. This will first be done for a steady-state model of the machine-convertor combination. Using the suppositions from this model, the dynamic model will be derived.

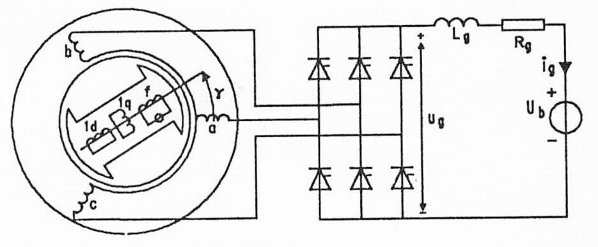

Figure 2 The system considered here

2 THE THREE-PHASE BRIDGE CONVERTOR

In the description of the convertor, the circuit shown in figure 3 will be used. The convertor is fed by a three-phase voltage source with internal self-inductance L_c and internal voltages e_a, e_b, and e_c according to

$$e_a=\hat{e}\cos(\omega t)\ ;\ e_b=\hat{e}\cos(\omega t-\frac{2}{3}\pi)\ ;\ e_c=\hat{e}\cos(\omega t-\frac{4}{3}\pi) \qquad (1)$$

where ω is a constant angular frequency and $\hat{e}$ is a constant amplitude. The convertor is loaded by a constant current source I_g. The thyristors will be considered as ideal switches; resistances in the circuit are neglected.

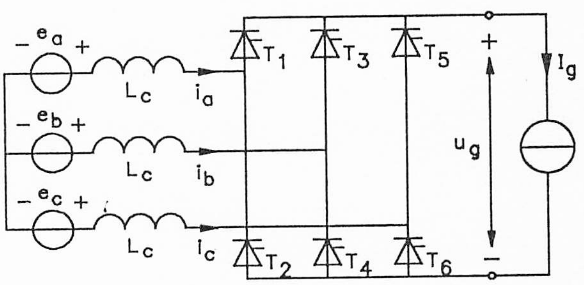

Figure 3 Base circuit for the convertor description

Each $\pi/3$ rad a thyristor is triggered. The convertor is controlled by varying the delay angle α: the angle by which the triggering instant is delayed with respect to the starting instant of the conduction of this thyristor in the case all thyristors are continuously triggered, i.e. the thyristors act like diodes. Hence a diode bridge rectifier corresponds with a convertor with $\alpha=0$.
Thanks to the symmetry of the circuit and of the currents and voltages in this circuit, the description of the convertor can be restricted to an interval of $\pi/3$ rad. Here the interval between the triggering instant of thyristor T_1 and the triggering instant of thyristor T_6 will be used: $-\pi/3+\alpha<\omega t<\alpha$. This interval is indicated by means of a thick line piece in figure 4. The angle of overlap μ, which will be defined later on, is supposed to be smaller than $\pi/3$ rad.
Just before the considered interval, the thyristors T_4 and T_5 are conducting; at the beginning of this interval thyristor T_1 will turn on and the current I_g starts to transfer from thyristor T_5 to thyristor T_1 (the starting instant of the commutation). During this commutation only the thyristors T_1, T_4, and T_5 are conducting. Hence, using (1) and the initial condition $i_a(-\pi/3+\alpha)=0$, the following relations can be given for the

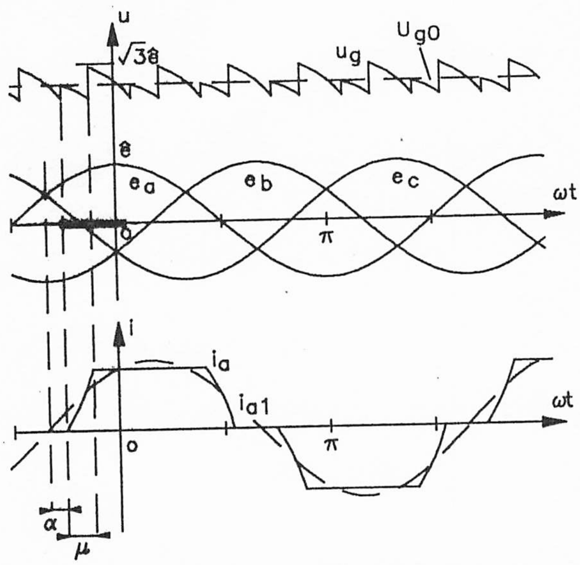

Figure 4 Some quantities as functions of ωt (α=0.3 ; μ=0.4)

commutation interval considered:

$$i_a = \frac{\sqrt{3}\hat{e}}{2\omega L_c}\{\cos\alpha - \cos(\omega t + \frac{\pi}{3})\} \ ; \ i_b = -I_g \tag{2a}$$

$$i_c = I_g - \frac{\sqrt{3}\hat{e}}{2\omega L_c}\{\cos\alpha - \cos(\omega t + \frac{\pi}{3})\} \ ; \ u_g = \frac{3}{2}\hat{e}\cos(\omega t + \frac{\pi}{3}) \tag{2b}$$

The commutation is finished when the current through thyristor T_5 (i_c) becomes zero. The time expressed in angular measure, elapsed from the beginning of the commutation until the end of the commutation is called the angle of overlap μ. In the considered interval the commutation is finished at the instant corresponding to $\omega t = -\pi/3 + \alpha + \mu$. From the condition $i_c(-\pi/3 + \alpha + \mu) = 0$ and (2), it follows:

$$\cos\alpha - \cos(\alpha + \mu) = \frac{2\omega L_c I_g}{\sqrt{3}\hat{e}} \tag{3}$$

After the commutation being finished, only the thyristors T_1 and T_4 are conducting. Using figure 3 and the voltage expressions (1), the following expressions can be given (only valid in the second part of the interval considered):

$$i_a = -i_b = I_g \ ; \ i_c = 0 \ ; \ u_g = e_a - e_b = \sqrt{3}\hat{e}\cos(\omega t + \frac{\pi}{6}) \tag{4}$$

The average value of the voltage u_g can be found by means of the expression (2), (4), and (3):

$$U_g = \frac{3}{\pi}\int_{\alpha - \frac{\pi}{3}}^{\alpha} u_g \, d\omega t = \frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - \frac{3}{\pi}\omega L_c I_g \tag{5}$$

By means of Fourier analysis and the equations (2), (3), and (4), the fundamental components of the phase currents may be expressed as

$$i_{a1}(\omega t) = i_{act}\cos(\omega t) \quad + i_{rea}\sin(\omega t) \tag{6a}$$

$$i_{b1}(\omega t) = i_{act}\cos(\omega t - \frac{2}{3}\pi) + i_{rea}\sin(\omega t - \frac{2}{3}\pi) \tag{6b}$$

$$i_{c1}(\omega t) = i_{act}\cos(\omega t - \frac{4}{3}\pi) + i_{rea}\sin(\omega t - \frac{4}{3}\pi) \tag{6c}$$

where the active and the reactive component coefficients are given by

$$i_{act} = \frac{\sqrt{3}}{\pi} I_g \{\cos\alpha + \cos(\alpha + \mu)\} \tag{7a}$$

$$i_{rea} = \frac{3\hat{e}}{2\omega L_c \pi}\{\mu - \sin\mu\cos(2\alpha + \mu)\} \tag{7b}$$

In many practical situations, the ripple on the direct current may be neglected, so that the above description may be used for the steady state. The description may also be used for slow changes in the amplitude or the frequency of the phase voltages and the average value of the direct current.
The dynamic model introduced in this way may be improved by enlarging the inductance in the dc-circuit with $2L_c$ [3]. This enlargement corresponds to the inductance seen from the dc-side of the convertor when two thyristors are conducting. Using (5), the equivalent circuit given in figure 5 may be composed.

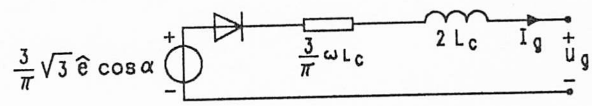

Figure 5 An equivalent circuit for the convertor

3 THE SYNCHRONOUS MACHINE

3.1 The basic set of equations

As in many cases, in this paper the (salient pole) synchronous machine is represented with one damper winding on the direct axis and one damper winding on the quadrature axis. In order to describe the machine, the Park transformation according to

$$i_d = \frac{\sqrt{2}}{\sqrt{3}}\{i_a\cos\gamma + i_b\cos(\gamma - \frac{2}{3}\pi) + i_c\cos(\gamma - \frac{4}{3}\pi)\} \tag{8a}$$

$$i_q = \frac{\sqrt{2}}{\sqrt{3}}\{i_a\sin\gamma + i_b\sin(\gamma - \frac{2}{3}\pi) + i_c\sin(\gamma - \frac{4}{3}\pi)\} \tag{8b}$$

$$i_0 = \frac{1}{\sqrt{3}}\{i_a + i_b + i_c\} \tag{8c}$$

will be used. The angle γ is defined in figure 2. For the phase voltages and flux linkages similar formulas will be used. As may be seen in figure 2, the homopolar current is zero. For that reason no attention is paid to this component.
With the usual suppositions for synchronous machines (see e.g. [4]), the stator voltage equations are given by ($\omega = d\gamma/dt$)

$$u_q = -R_a i_q - \frac{d\psi_q}{dt} + \omega\psi_d \tag{9a}$$

$$u_d = -R_a i_d - \frac{d\psi_d}{dt} - \omega\psi_q \tag{9b}$$

The rotor voltage equations are

$$0 = R_{1q} i_{1q} + \frac{d\psi_{1q}}{dt} \tag{10a}$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \tag{10b}$$

$$0 = R_{1d} i_{1d} + \frac{d\psi_{1d}}{dt} \tag{10c}$$

In these expressions, the subscripts 1q, f, and 1d refer to the damper winding on the quadratue axis, the excitation (field) winding, and the damper winding on the direct axis respectively.
The flux linkages in the voltage equations are given by

$$\psi_q = L_q i_q + L_{a1q} i_{1q} \tag{11a}$$

$$\psi_{1q} = L_{a1q} i_q + L_{11q} i_{1q} \tag{11b}$$

$$\psi_d = L_d i_d + L_{afd} i_f + L_{a1d} i_{1d} \tag{12a}$$

$$\psi_f = L_{afd} i_d + L_f i_f + L_{f1d} i_{1d} \tag{12b}$$

$$\psi_{1d} = L_{a1d} i_d + L_{f1d} i_f + L_{11d} i_{1d} \tag{12c}$$

The electromagnetic torque may be found by

$$m = i_d\psi_q - i_q\psi_d \tag{13}$$

It should be noted that the per-unit system will not be used in this paper.

3.2 Referring the damper quantities to the stator

Since the damper circuits are not directly accessible, the real values of the damper quantities are not of interest (from a system point of view) and it makes sense to refer them to the stator.

Since the rotor winding fluxes may be considered as constant for very fast phenomena like the commutation in a convertor (they are "short-circuited"), it is advantageous to use these fluxes as state variables in the machine model for the combination synchronous machine with convertor. As we shall see later on, the supposition that the rotor fluxes may be seen as constant for very fast phenomena corresponds with the observation that very fast changing currents i_q and i_d see the subtransient inductances only. Hence, it is practical to refer the damper quantities to the stator in such a way, that the subtransient inductances become explicitly visible.

The quadrature axis

The quadrature-axis damper quantities are referred to the stator by introducing a new set of damper quantites

$$\psi_{1Q} = C_{1Q}\,\psi_{1q} \;;\; i_{1Q} = \frac{1}{C_{1Q}}\,i_{1q} \tag{14}$$

where C_{1Q} is a factor which may freely be chosen.

In order to make the subtransient inductance explicitly visible, C_{1Q} is defined according to [5]

$$C_{1Q} = \frac{L_{a1q}}{L_{11q}} \tag{15}$$

After introducing the quantities

$$L_q'' = L_q - \frac{L_{a1q}^2}{L_{11q}} \;;\; L_{1Q} = \frac{L_{a1q}^2}{L_{11q}} \;;\; R_{1Q} = \frac{L_{a1q}^2}{L_{11q}^2}R_{11q} \tag{16}$$

the quadrature-axis equations (9a), (10a), and (11) become

$$u_q = -R_a i_q - \frac{d\psi_q}{dt} + \omega\psi_d \tag{17a}$$

$$0 = R_{1Q}i_{1Q} + \frac{d\psi_{1Q}}{dt} \tag{17b}$$

$$\psi_q = L_q'' i_q + L_{1Q}(i_q + i_{1Q}) \tag{18a}$$

$$\psi_{1Q} = L_{1Q}(i_q + i_{1Q}) \tag{18b}$$

By means of this set of equations, the equivalent circuit shown in figure 6 may be constructed. As may be seen in this figure, the inductance L_{1Q} is short-circuited by the resistance R_{1Q} for very fast changing currents i_q. So, these very fast changing currents only "see" L_q'', and the flux ψ_{1Q} may be considered as constant. Hence, L_q'' is the quadrature-axis subtransient inductance.

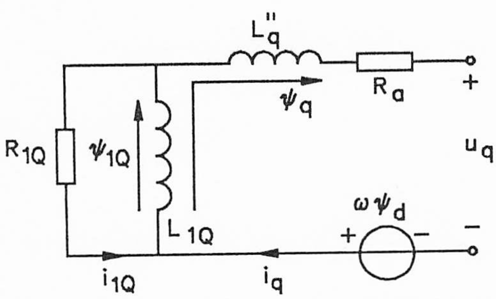

Figure 6 An equivalent circuit for the quadrature axis with the subtransient inductance explicitly present.

The quantities i_q and ψ_{1Q} will be used as state variables for the quadrature axis. After eliminating i_{1Q} and ψ_q in (17) and (18b) and i_{1Q} in (18a), the equations (17) and (18a) may be written as

$$u_q = -(R_a + R_{1Q})i_q - L_q''\frac{di_q}{dt} + \frac{R_{1Q}}{L_{1Q}}\psi_{1Q} + \omega\psi_d \tag{19a}$$

$$\frac{d\psi_{1Q}}{dt} = -R_{1Q}\left\{\frac{\psi_{1Q}}{L_{1Q}} - i_q\right\} \tag{19b}$$

$$\psi_q = L_q'' i_q + \psi_{1Q} \tag{19c}$$

The direct axis

The direct-axis damper quantities are referred to the stator by introducing the new set of damper quantities

$$\psi_{1D} = C_{1D}\,\psi_{1d} \;;\; i_{1D} = \frac{1}{C_{1D}}\,i_{1d} \tag{20}$$

where C_{1D} is a factor which may freely be chosen.

In order to make the subtransient inductance explicitly visible, C_{1D} is defined according to [5]

$$C_{1D} = \frac{L_{a1d}}{L_{11d}} \tag{21}$$

Besides, the parameters

$$L_{1D} = C_{1D}^2\,L_{11d} = \frac{L_{a1d}^2}{L_{11d}} \;;\; K_{f1D} = \frac{L_{f1d}}{L_{a1d}} \;;$$

$$L_f' = L_f - K_{f1D}^2 L_{1D} = L_f - \frac{L_{f1d}^2}{L_{11d}} \;;\; C_F = \frac{L_{afd} - K_{f1D}L_{1D}}{L_f'} \;;$$

$$L_d'' = L_d - L_{1D} - C_F^2 L_f' \;;\; R_{1D} = C_{1D}^2 R_{11d} \;; \tag{22}$$

are introduced. Now, the direct axis equations (9b), (10b), (10c), and (12) may be written as

$$u_d = -R_a i_d - \frac{d\psi_d}{dt} - \omega\psi_q \tag{23a}$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \tag{23b}$$

$$0 = R_{1D}i_{1D} + \frac{d\psi_{1D}}{dt} \tag{23c}$$

$$\psi_d = L_d'' i_d + C_F L_f'(C_F i_d + i_f) + L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \tag{24a}$$

$$\psi_f = L_f'(C_F i_d + i_f) + K_{f1D}L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \tag{24b}$$

$$\psi_{1D} = L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \tag{24c}$$

These equations may be used for the construction of the circuit in figure 7. It should be noted that the transformers indicated with $1{:}C_F$ and with $K_{f1D}{:}1$ in this figure are ideally coupled (without leakage). The quantities i_d, ψ_{1D}, and

$$\psi_f' = L_f'(C_F i_d + i_f) \tag{25}$$

will be used as state variables for the direct axis. After eliminating i_f, i_{1D}, ψ_f, and ψ_d in (23), (24b), (24c), and (25) and i_f and i_{1D} in (24a), the equations (23) and (24a) may be written as

$$\begin{aligned} u_d = &-\{R_a + C_F^2R_f + (1 - C_FK_{f1D})^2R_{1D}\}i_d - L_d''\frac{di_d}{dt} - C_Fu_f \\ &-\frac{\psi_f'}{L_f'}\{-C_FR_f + K_{f1D}(1 - C_FK_{f1D})R_{1D}\} + \psi_{1D}\frac{R_{1D}}{L_{1D}}(1 - C_FK_{f1D}) - \omega\psi_q \end{aligned} \tag{26a}$$

$$\begin{aligned} \frac{d\psi_f'}{dt} = &\; i_d\{C_FR_f - K_{f1D}R_{1D}(1 - C_FK_{f1D})\} + u_f \\ &-\frac{\psi_f'}{L_f'}(R_f + K_{f1D}^2R_{1D}) + K_{f1D}\frac{R_{1D}}{L_{1D}}\psi_{1D} \end{aligned} \tag{26b}$$

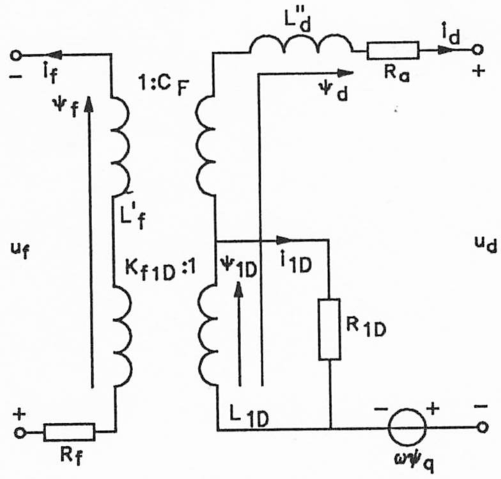

Figure 7 An equivalent circuit for the direct axis with the subtransient inductance explicitly present

$$\frac{d\psi_{1D}}{dt} = R_{1D}(1-C_FK_{f1D})i_d + K_{f1D}R_{1D}\frac{\psi_f'}{L_f'} - \frac{R_{1D}}{L_{1D}}\psi_{1D} \tag{26c}$$

$$\psi_d = L_d''i_d + C_F\psi_f' + \psi_{1D} \tag{26d}$$

3.3 The internal voltages

Substituting (26d) into (19a) and (19c) into (26a), the equations (19a) and (26a) may be written as

$$u_q = -(R_a+R_{1Q})i_q - L_q''\frac{di_q}{dt} + \frac{R_{1Q}}{L_{1Q}}\psi_{1Q} + \omega(L_d''i_d+C_F\psi_f'+\psi_{1D}) \tag{27a}$$

$$\begin{aligned} u_d = &-\{R_a+C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}\}i_d - L_d''\frac{di_d}{dt} \\ &- \frac{\psi_f'}{L_f'}\{-C_FR_f+K_{f1D}(1-C_FK_{f1D})R_{1D}\} \\ &+ \psi_{1D}\frac{R_{1D}}{L_{1D}}(1-C_FK_{f1D}) - C_Fu_f - \omega(L_q''i_q+\psi_{1Q}) \end{aligned} \tag{27b}$$

When the armature currents and voltages are changing very rapidly (such as in case of commutation in the rectifier; with constant u_f and ω), the fluxes in the damper windings and in the excitation winding may be considered to be constant in first instance (the rotor windings are "short-circuited"). Hence, the voltage terms in the equations (27) which depend on the fluxes ψ_{1Q}, ψ_f ($=\psi_f'+K_{f1D}\psi_{1D}$), and ψ_{1D} may be seen as constant for this case, and it might be useful to introduce the internal voltages

$$e_q = \frac{R_{1Q}}{L_{1Q}}\psi_{1Q} + \omega(C_F\psi_f'+\psi_{1D}) \tag{28a}$$

$$\begin{aligned} e_d = &-\frac{\psi_f'}{L_f'}\{-C_FR_f+K_{f1D}(1-C_FK_{f1D})R_{1D}\} + \psi_{1D}\frac{R_{1D}}{L_{1D}}(1-C_FK_{f1D}) \\ &- C_Fu_f - \omega\psi_{1Q} \end{aligned} \tag{28b}$$

So, these voltages are constant in case of very rapidly changing currents i_d and i_q (constant ω and u_f).
Besides, we introduce the resistances

$$R_q'' = R_a + R_{1Q} \tag{29a}$$

and

$$R_d'' = R_a + C_F^2R_f + (1-C_FK_{f1D})^2R_{1D} \tag{29b}$$

These resistances are seen from the armature when the armature currents are changing very rapidly (with ω, e_q and e_d considered constant).
Using (28) and (29), the equations (27) may be written as

$$u_q = e_q - R_q''i_q - L_q''\frac{di_q}{dt} + \omega L_d''i_d \tag{30a}$$

$$u_d = e_d - R_d''i_d - L_d''\frac{di_d}{dt} - \omega L_q''i_q \tag{30b}$$

3.4 Adaptation of the model for the convertor

When modelling the synchronous machine for the case it is connected with a convertor, it is practical to transform a number of Park quantities (*dq*) back to the armature reference system (*abc*). At first, this results in intricate expressions, but these may be simplified.
Using the Park transformation for the currents (8) and the similar expressions for the voltages, the equations (30) may be transformed back to the armature reference system (the homopolar components have been supposed to be zero; ω=dγ/dt):

$$\begin{aligned} u_a = &\frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos\gamma + e_q\sin\gamma\} - \frac{R_d''+R_q''}{2}i_a - \frac{L_d''+L_q''}{2}\frac{di_a}{dt} \\ &+ \frac{2}{3}\omega(L_d''-L_q'')\{\sin(2\gamma)i_a+\sin(2\gamma-\tfrac{2}{3}\pi)i_b+\sin(2\gamma-\tfrac{4}{3}\pi)i_c\} \\ &- \frac{R_d''-R_q''}{3}\{\cos(2\gamma)i_a+\cos(2\gamma-\tfrac{2}{3}\pi)i_b+\cos(2\gamma-\tfrac{4}{3}\pi)i_c\} \\ &- \frac{L_d''-L_q''}{3}\{\cos(2\gamma)\frac{di_a}{dt}+\cos(2\gamma-\tfrac{2}{3}\pi)\frac{di_b}{dt}+\cos(2\gamma-\tfrac{4}{3}\pi)\frac{di_c}{dt}\} \end{aligned} \tag{31}$$

The expressions for u_b and u_c look similar.
Fortunately, these equations may be simplified for most practical situations. The first simplification is to neglect the terms with $L_d''-L_q''$ with respect to the terms with $L_d''+L_q''$. The second simplification is to neglect the resistive terms with respect to the inductive terms. Using these simplifications, (31) becomes (extended with the expressions for u_b and u_c)

$$u_a = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos\gamma + e_q\sin\gamma\} - \frac{L_d''+L_q''}{2}\frac{di_a}{dt} \tag{32a}$$

$$u_b = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{2}{3}\pi) + e_q\sin(\gamma-\tfrac{2}{3}\pi)\} - \frac{L_d''+L_q''}{2}\frac{di_b}{dt} \tag{32b}$$

$$u_c = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{4}{3}\pi) + e_q\sin(\gamma-\tfrac{4}{3}\pi)\} - \frac{L_d''+L_q''}{2}\frac{di_c}{dt} \tag{32c}$$

These expressions are depicted schematically in figure 8.

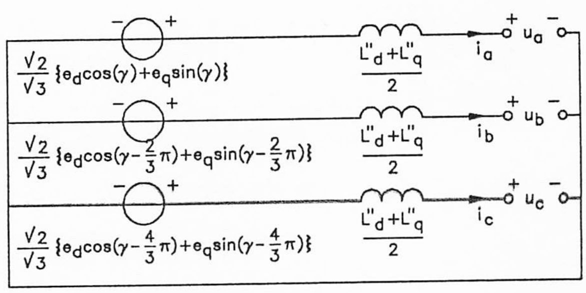

Figure 8 The simplified armature circuit

3.5 Summary

Using (19c) and (26d), the expression for the torque (13) may be written as

$$m = i_d(L_q'' i_q + \psi_{1Q}) - i_q(L_d'' i_d + C_F \psi_f' + \psi_{1D}) \qquad (33)$$

Now, the simplified model of the synchronous machine is described by the equations (8), (19b), (26b), (26c), (28), (32), and (33). The resistive terms and the terms with $L_d''-L_q''$ have only been neglected in equation (32).

4 THE SYNCHRONOUS MACHINE WITH CONVERTOR

4.1 The steady-state model

In the previous chapters, the convertor and the synchronous machine have been treated separately. However, the models developed in these chapters cannot simply be connected. Considering figure 9, in this section a method will be given to develop a steady-state model of the combination.

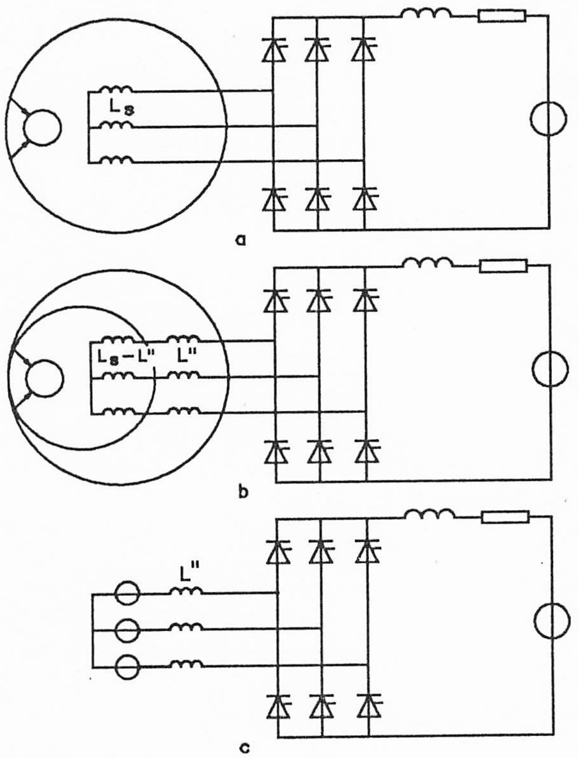

Figure 9 Splitting off the subtransient inductance

In order to investigate the interaction between the machine and the convertor, we shall consider the harmonics in the phase currents. In the system considered, the armature currents may be expressed as Fourier series in which all even harmonics are zero thanks to the property $i(\omega t-\pi)=-i(\omega t)$, where ω represents the angular speed of the rotor. Moreover, as the star connection terminal of the machine is not used, the armature phase currents do not contain harmonics with an angular frequency which is a multiple of 3ω. Hence, the Fourier series consists of a fundamental component with angular frequency ω and harmonics with angular frequencies of $(6k-1)\omega$ and $(6k+1)\omega$, where k is an integer larger than 0.
Since the rotor "sees" the fundamental components of the phase currents as direct currents, they don't induce currents in the rotor circuits. Hence, the impedance of the stator for the fundamental components is determined by the synchronous inductances. Seen from the rotor, the harmonics in the armature currents are transformed into currents with angular frequencies of $6k\omega$. In practice, these are relatively very high frequencies, so that these harmonics in the armature currents hardly cause changes in the rotor fluxes ψ_{1Q}, ψ_f, and ψ_{1D}. As has been explained in chapter 3, this means that the voltages e_d and e_q may be considered constant, so that the voltage sources in figure 8 are sinusoidal. Hence, the armature current harmonics only see the subtransient inductance $L''=(L_q''+L_d'')/2$.
This inductance is explicitly splitted off by subtracting it from the synchronous inductances (see figures 9a and 9b). Here arises the so-called internal machine: the original machine minus the subtransient inductances $L''=(L_q''+L_d'')/2$. This is a normal synchronous machine, with the exception that it is a short-circuit for the armature phase current harmonics. This means that the armature phase voltages of this internal machine are always sinusoidal. They only depend on the excitation current and the fundamental components of the armature phase currents (or the dc-components of i_d and i_q).
Hence, the internal machine may be represented by a sinusoidal three-phase voltage source (figure 9c), which is controlled by the excitation current and the fundamental components of the armature phase currents.
Using figure 8, these voltage sources may be described by

$$e_a = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma)+e_q\sin(\gamma)\} \qquad (34a)$$

$$e_b = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{2}{3}\pi)+e_q\sin(\gamma-\tfrac{2}{3}\pi)\} \qquad (34b)$$

$$e_c = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{4}{3}\pi)+e_q\sin(\gamma-\tfrac{4}{3}\pi)\} \qquad (34c)$$

Choosing

$$\gamma = \omega t + \frac{\pi}{2} \qquad (35)$$

these equations may be written as

$$e_a = \hat{e}\cos(\omega t-\varepsilon) \qquad (36a)$$

$$e_b = \hat{e}\cos(\omega t-\varepsilon-\tfrac{2}{3}\pi) \qquad (36b)$$

$$e_c = \hat{e}\cos(\omega t-\varepsilon-\tfrac{4}{3}\pi) \qquad (36c)$$

where

$$\hat{e} = \frac{\sqrt{2}}{\sqrt{3}}\sqrt{e_d^2+e_q^2} \qquad (37a)$$

$$\varepsilon = -\arctan\frac{e_d}{e_q} \qquad (37b)$$

Figure 9c corresponds with figure 3 and the only difference between the expressions (36) and the expressions (1) is the phase shift ε, which represents the load angle of the internal machine of figure 9b. Besides, we have to choose

$$L_c = \frac{L_d'' + L_q''}{2} \qquad (38)$$

In order to compute the steady state of the synchronous machine with convertor, the constant parts of i_d and i_q have to be known. These constant parts correspond to the basic harmonics of the armature phase currents, so that they may be found by substituting (6) with ωt replace by $\omega t-\varepsilon$ into (8). Next, using (35) results in

$$i_d = -\frac{\sqrt{3}}{\sqrt{2}}\{\hat{i}_{act}\sin\varepsilon + \hat{i}_{rea}\cos\varepsilon\} \qquad (39a)$$

$$i_q = \frac{\sqrt{3}}{\sqrt{2}}\{\hat{i}_{act}\cos\varepsilon - \hat{i}_{rea}\sin\varepsilon\} \qquad (39b)$$

The set of equations (3), (5), (7), (19b), (26b), (26c), (28), (33), (37), (38), and (39) with $d\psi_f'/dt=d\psi_{1D}/dt=d\psi_{1Q}/dt=0$ gives a complete description of the steady-state model of the synchronous machine with convertor. This set may be solved numerically.

4.2 The dynamic model

In the dynamic model of the system, the description of the convertor in chapter 2 will be used (the ripple on the current in the dc-link is neglected). Combining the model from this chapter (figure 5) and figure 2 with (38) results in an equation for the dc-circuit (adding the resistance R_g and the voltage source U_b from figure 2 to figure 5):

$$\frac{di_g}{dt} = \frac{\frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - \left(\frac{3}{\pi}\omega\frac{L_d''+L_q''}{2}+R_g\right)i_g - U_b}{L_g + 2L_c} \tag{40}$$

As in section 4.1, only the fundamental components of the phase currents of the synchronous machine are taken into account to model the synchronous machine with convertor. However, the amplitude, phase and angular frequency of the basic harmonics may vary now. To allow use of the description of the convertor in chapter 2, these variations should be slow compared to the commutation phenomena.
As has been explained in section 4.1, the fundamental components of the phase currents are transformed to dc-components in the currents i_d and i_q. In the machine model only these dc-components are considered. When the basic harmonics vary "slowly", these "dc" components will vary slowly too. These "dc" components, resembling short-term averages, will be used in the machine model.
In the steady-state case, we only consider the dc-components in i_d and i_q and neglect the components with an angular frequency which is an integer multiple of 6ω. Hence, the angular frequency of the variation of the "dc" component should be much smaller than 6ω. So, if, for example, the frequency of the basic component of the phase current equals 50 Hz, the frequency of the variation of this basic harmonic should be much smaller than 300 Hz.
Now, we have found a fourth order model of the synchronous machine with convertor. This model is described by the set of equations (3), (7), (19b), (26b), (26c), (28), (33), (37), (39) and (40). This is a fourth order model with i_g, ψ_f, ψ_{1D}, and ψ_{1Q} as state variables.

CONCLUSION

In this paper a rather simple fourth-order dynamic model of a synchronous machine with dc-link is presented. For the case where the convertor consists of diodes and the synchronous machine has two damper windings on the direct axis, the results obtained by means of this simple model have been compared with measurements [6]. It appeared, that using the simple model presented here comes up to the expectations: the short-term averaged values of the system variables are simulated correctly; details, such as the harmonics on the phase currents and the ripple on the direct current are not taken into acccount.
At the moment of writing this paper, research attention was aimed at the experimental validation of the model presented here (with one damper winding on the direct axis and a thyristor bridge convertor).

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