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A PRACTICAL METHOD OF DETERMINING QUADRATURE-AXIS SYNCHRONOUS MACHINE PARAMETERS USING A LEAST QUARES ESTIMATOR

VLEESHOUWERS, J.M. - HOEIJMAKERS, M.J. - SCHOT, J.A.
EINDHOVEN UNIVERSITY OF TECHNOLOGY, THE NETHERLANDS

ABSTRACT

The paper describes a practical off-line method of determining synchronous machine parameters using a least-squares estimator. The method involves terminal measurements at standstill. Any commonly available DC-power supply can be used as an excitation. Some estimation results are presented for a 4-parameter model of the quadrature axis of the synchronous machine.

1. INTRODUCTION

The determination of model parameters is an essential aspect of almost any engineering effort. In the case of synchronous machine parameters it is often also a cumbersome aspect. A number of reasons can be given, but the most important is, that the measurement set-up, which is needed to determine the parameters, often requires expensive equipment or time-consuming preparations. For this reason most parameter determination methods are either impractical or inflexible. In this paper a method will be presented which tries to overcome this disadvantage. It is based on the standstill step-response test and on modern estimation techniques for off-line parameter determination.
The necessary measurement set-up only consists of a DC-power supply (or battery), which generates an approximate step-excitation, and a signal recorder to record current-response and voltage-excitation. The machine is at standstill. The exact characteristics of the excitation signal generator are not a point of concern. This is the main difference with parameter estimation schemes proposed by Ritter, Shackshaft, Irisa and Watson [1,2,3,4]: these schemes require excitation sources which meet critical specifications. From the recorded measurements the parameters are calculated using a least-squares estimator algorithm, which is implemented on a personal computer.

In the next paragraphs we shall first describe the background of the proposed estimation scheme. The method will then be clarified using a simple example: a 4-parameter model of the quadrature axis of the machine. The paper will focus on the model, the measurement set-up, and on a number of estimations. The estimation results are discussed and compared to previous measurements of the parameters. Since this paper only aims at presenting the method outlined above, no other validation of the estimates has been attempted. One may, however, expect the validity to be similar to that of other parameter determination techniques which are based on step responses.

2. ESTIMATION CONCEPT

The problem of designing a method for parameter determination can be formulated concisely using systems theory concepts. A detailed analysis of these concepts can be found in many handbooks on this subject (e.g. [5]). Let us consider a system at which we can perform input-output measurements. Suppose we have a model, which describes the input-output relations by

$$0 = f(u,y,\underline{\theta}) \qquad (1)$$

where u and y represent the input signal and the output signal respectively as a function of time, and $\underline{\theta}$ represents the parameters. The function f is called the model-function. In case of a linear model f is a linear differential equation.
If we excite the system (physically) with the input $u_0 = u_0(t)$, we can measure the response of the system, the output $y_0 = y_0(t)$.
Substituting $u_0(t)$ and $y_0(t)$ into (1) results in

$$0 = f(u_0,y_0,\underline{\theta}) \qquad (2)$$

which is an equation from which $\underline{\theta}$ can be solved in principle. In practical situations ther will be measurement errors, noise and model imperfections, so that there will be no $\underline{\theta}$ which satisfies (2) for all instants. In that case only such a $\underline{\theta}$ can be determined as to satisfy (2) "best". To determine this $\underline{\theta}$ mathematically, an error criterion is needed. This criterion is formulated using a scalar function E of $\underline{\theta}$, u_0 and y_0, at the minimum of which $\underline{\theta}$ has by definition its "best" value. We shall call this "best" value $\underline{\theta}_0$.

Usually, u_0 and y_0 are given as a set of samples at K successive instants t_1 to t_K. Denoting the k-th sample of u_0 and y_0 as $u_0(k)$ and $y_0(k)$, we may define the loss-function E in a least-squares sense by:

$$E = \sum_{k=1}^{K} e_k{}^2 \qquad (3)$$

where the K functions $e_k(\underline{\theta})$, the so called residual functions, are given by:

$$e_k(\underline{\theta}) = f(u_0(k), y_0(k), \underline{\theta}) \qquad (4)$$

A minimization of (3) can be obtained using well known numerical minimization techniques.

Concluding from the previous discussion, the next three elements will be essential components of a successful parameter estimation:

- A model, which is fit to describe the system behaviour in the considered situation.
- An appropriate input signal and a practical way of supplying the input signal and measuring the output signal.
- The residual functions, and an algorithm to minimize the error criterion with respect to the parameters.

These elements will underly the next chapters, in which we shall focus on an example: the q-axis parameters of a synchronous machine.

3. MODEL OF QUADRATURE AXIS

The model which is used in the following parameter estimation example, describes the quadrature axis of a linearly magnetic three phase machine which has one damper winding on this axis. For a detailed description of the background of this model, we refer to textbooks on the synchronous machine (e.g. [6]).

A network representation of the quadrature-axis model is drawn in figure 1. The classical representation with stator and rotor leakage inductances is transformed into a four-parameter network using well known reduction techniques from transformer theory [7]. In this network the subtransient inductance L''_q and the armature phase resistance R_a can be distinguished. The synchronous inductance L_q is equal to the sum of L''_q and L_{aQ}.

The flux ψ_d originates from the direct axis, and is not important since we deal with standstill tests for which ω, the angular velocity of the rotor, is zero.

The actual stator voltages u_a, u_b and u_c relate to u_q via the Park-transformation:

$$u_q = \sqrt{\tfrac{2}{3}}\left[u_a \sin(\gamma) + u_b \sin(\gamma - \tfrac{2}{3}\pi) + u_c \sin(\gamma - \tfrac{4}{3}\pi)\right] \qquad (5)$$

The same holds for the set of armature currents i_a, i_b and i_c and their relation to i_q:

$$i_q = \sqrt{\tfrac{2}{3}}\left[i_a \sin(\gamma) + i_b \sin(\gamma - \tfrac{2}{3}\pi) + i_c \sin(\gamma - \tfrac{4}{3}\pi)\right] \qquad (6)$$

In (5) and (6) the angle γ is the position of the rotor direct-axis with respect to the axis of the stator a-winding.

The network of figure 1 contains 4 parameters: L''_q, L_{aQ}, R_a, and R_Q. The model equation (7) relates these parameters to u_q and i_q:

$$0 = u_q(t) + \frac{L_{aQ}}{R_Q}\frac{du_q(t)}{dt} + R_a i_q(t) + \left(L''_q + L_{aQ} + R_a\frac{L_{aQ}}{R_Q}\right)\frac{di_q(t)}{dt} + L''_q\frac{L_{aQ}}{R_Q}\frac{d^2 i_q(t)}{dt^2} \qquad (7)$$

This relation corresponds to equation (1) with $\underline{\theta} = (R_a, L''_q, L_{aQ}, R_Q)$, $u = u_q$ and $y = i_q$.

4. THE MEASUREMENT SET-UP

An important goal in the design of the estimation scheme was to ensure a practical measurement set-up: it should use none but the most elementary instruments and techniques. Considering this, the measurements were chosen as follows:

- the response of the machine is measured at standstill;
- only terminal measurements of voltage and current are carried out.

As far as measuring equipment is concerned, we demanded portability, compactness, common availability and ease of use.

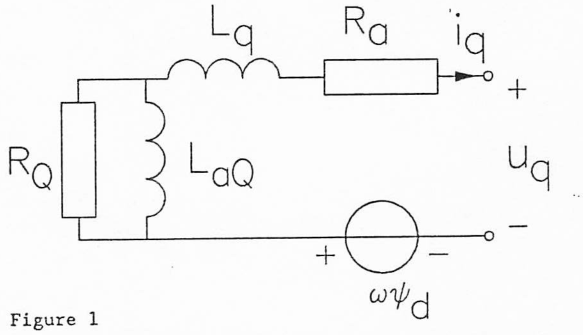

Figure 1
Two winding model of q-axis of synchronous machine

4.1. THE EXCITATION SIGNAL

The generation of the excitation signal required some special consideration. This signal needs to be persistently exciting (i.e. the excitation and response contain sufficient information for a succesful estimation), exactly known and also easy to generate. An approximate step excitation appears to be the best way to combine these demands.
From previous experiments it is known that an ideal voltage step excitation, applied to the terminals, is persistently exciting [2]. An ideal step (i.e. with infinitely steep edge and constant amplitude after swith-on) cannot be generated, but a non-ideal step is quite easy to generate. If it is measured and registered along with the response of the system, it is also exactly known. Finally, we may expect to gather sufficient information for an estimation provided the excitation does not essentially differ from an ideal step.
By recording the excitation, the estimation also attains the desirable property of becoming relatively independent on the choice of the signal source, which further facilitates its design.
In our experiments a battery has been used as a signal source. After S has been closed the thyristor T is triggered. Diode D, the free-wheeling diode, is included in the circuit to prevent damage from high voltages which are induced when the current i is switched off.

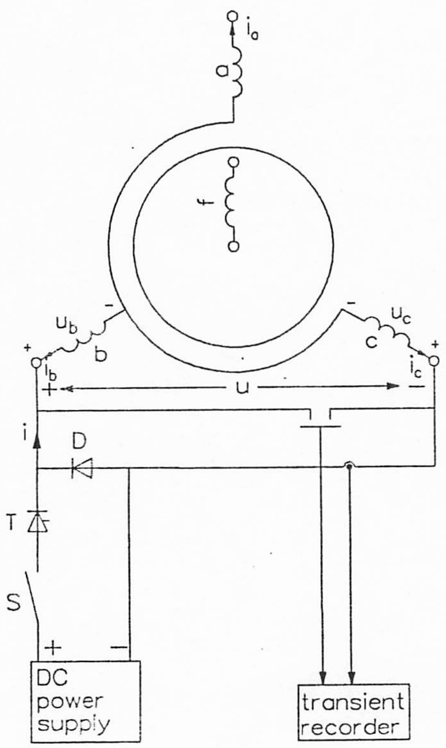

Figure 2
Measurement set-up

4.2. THE MEASUREMENT SET-UP

The measurement set-up is depicted in figure 2. From figure 2 it follows that

$$i_a = 0 \tag{8a}$$
$$u_b - u_c = u \tag{8b}$$

If the rotor is at the position shown (γ=0), from (8) and the Park transformation (5) and (6) the relation between the quantities u, u_q, i, and i_q can be derived:

$$u_q = -\frac{1}{2}\sqrt{2}\, u \tag{9a}$$
$$i_q = \sqrt{2}\, i \tag{9b}$$

A zero flux current transformer is used to measure the current i. A transient recorder is used to measure and record the output of the current transformer and the voltage u. The signals are recorded with a sample rate of 2 ms during 1.5 s. After a recording has been completed, the current is switched off and the data is stored on floppy disk using a personal computer. Figure 3 shows a typical u_q and i_q as a function of time.

4.3. THE SYNCHRONOUS MACHINE

Using this measurement set-up, estimations have been carried out at a synchronous machine which was available in our laboratory: a 24 kVA three-phase inverted machine (with a four pole excitation winding on the stator and armature windings on the rotor). The machine has a cage damper winding consisting of solid cylindrical conductors lying in semi-closed slots in the laminated stator.
Because of the fact that the dampers of the model are simple R-L-circuits, whereas the real damper is more complex (e.g. due to skin-effect), some difficulties are to be expected in fitting the model to the system. Estimation ambiguities may also arise because of the fact that the model does not account for saturation and hysteresis.

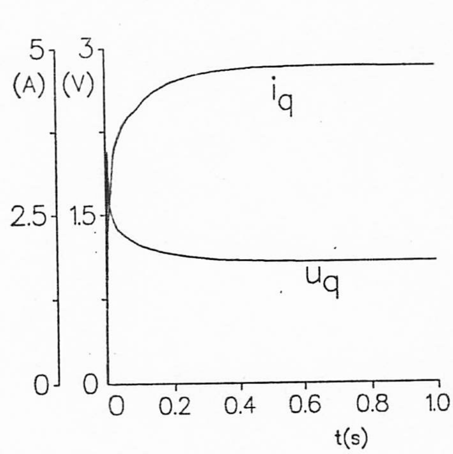

Figure 3
Typical u_q and i_q as a function of time

5. THE RESIDUAL FUNCTIONS

Since the determination of the quadrature-axis parameters only serves as an example, an essential part of the estimation is lacking: the context in which the parameters are going to be used. This context is needed to validate estimation results and can also indicate the choice of the residual function.

We shall use here, rather arbitrarily, residual fuctions $e_k(\underline{\theta})$ $(1 \le k \le K)$, which follow from the model equation (7) by integrating once over t, multiplying by R_Q/L_{aQ} and substituting the instances t_k for t:

$$e_k = \frac{R_Q}{L_{aQ}} \int_{-\infty}^{t_k} u_q(\tau)d\tau + u_q(t_k) + \frac{R_Q}{L_{aQ}} R_a \int_{-\infty}^{t_k} i_q(\tau)d\tau$$

$$+ (R_a + R_Q + L''_q \frac{R_Q}{L_{aQ}})\ i_q(t_k) + L''_q \left(\frac{di_q(t)}{dt}\right)\Big|_{t=t_k} \qquad (10)$$

Defining:

$$a_{1k} = \int_{-\infty}^{t_k} u_q(\tau)d\tau \qquad (11a)$$

$$a_{2k} = u_q(t_k) \qquad (11b)$$

$$a_{3k} = \int_{-\infty}^{t_k} i_q(\tau)d\tau \qquad (11c)$$

$$a_{4k} = i_q(t_k) \qquad (11d)$$

$$a_{5k} = \frac{di_q(t)}{dt}\Big|_{t=t_k} \qquad (11e)$$

the residual functions become:

$$e_k = \frac{R_Q}{L_{aQ}} a_{1k} + a_{2k} + \frac{R_Q}{L_{aQ}} R_a\ a_{3k} +$$

$$(R_a + R_Q + \frac{R_Q}{L_{aQ}} L''_q)\ a_{4k} + L''_q\ a_{5k} \qquad (12)$$

The coefficients a_{ik} follow from the measured data, either directly (a_{2k}, a_{4k}) or indirectly (a_{1k}, a_{3k}, a_{5k}). Various numerical methods may be applied to approximate the integrals and the derivative. (See e.g. [8].)

After having computed the coefficients a_{ik}, substitution results in K equations in the four parameters R_a, L''_q, R_Q and L_{aQ}. The loss function (3) can now be minimized with respect to these parameters. For reasonable results K should be considerably larger than 4. In our case the Marquardt algorithm was chosen to minimize E [9]. This algorithm is known to be a fair compromise between robustness and speed. Like all non-linear minimization algorithms, it needs more or less accurate parameters values to initialise the iteration.

6. ESTIMATION RESULTS

In this section a number of estimation results will be presented. We shall focus on two model-inaccuracies (those due to skin-effect and hysteresis) and try to point out how they affect the estimation. The estimation results are compared to results of previous measurements at the same machine.

6.1. SKIN-EFFECT

The skin-effect of the cage damper of the considered machine causes the 4-parameter model to be imperfect. The inductance and resistance of the damper are a function of the time after the step excitation has been applied. Initially the inductance is lower and the resistance is higher than after some time has elapsed.

The first series of estimations from the measured data was aimed at checking the influence of the skin-effect on the estimation. Since the skin-effect plays a role just after switch-on, estimations were carried out on 4 subsets of the measured data, from which the first few samples had been excluded: if we specify the chosen instants by:

$$t_k = t_1 + (k-1)\ T, \quad 1 \le k \le K \qquad (13)$$

with t_1 the first chosen instant after the instant of triggering and T the sampling period; t_1 was varied from 2 to 16 ms, T was fixed at 4 ms and K was set to have the last instant at 0.6 s.

Figure 4 shows the estimation results and figure 5 shows the residual $e(\underline{\theta})$, computed at the estimated parameter values. A clear trend is noticable: if the starting instant t_1 increases, the resulting residual in the first 0.1 s becomes smaller. This can be explained by realizing that the first instants after switch-on contain information which cannot be accounted for by the model. Choosing a smaller t_1 will thus result in a worse in the interval just after triggering.

Considering this, a decision on which choice of t_1 will give the best estimation results will be a compromise and depend on the specific use of the parameters.

6.2. HYSTERESIS

To check the influence of hysteresis, seven successive measurements were carried out. They took place within half an hour. The sign of the excitation was varied in the order 1:+ (positive), 2:+, 3:- (negative), 4:-, 5:+, 6:+ and 7:+. Measurement pairs 2 and 3 as well as 4 and 5 were

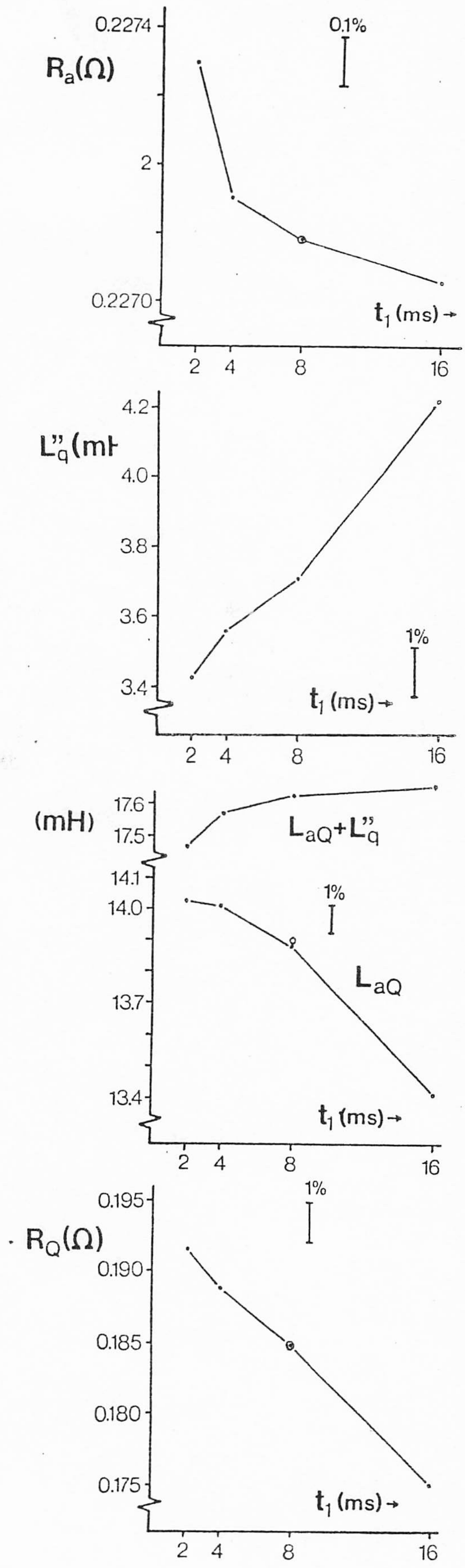

Figure 4

Influence of skin-effect on estimation results

inserted to allow hysteresis to influence the estimation results. For measurements 3 and 5 the effective part of the hysteresis loop will be steeper than for the measurements 2 , 4, 6 and 7. Therefore the synchronous inductance L_q $(=L_{aQ}+L''_q)$ will be higher.

As before, a residual set with t_1=2ms, T=4ms and K=150 was chosen to estimate from the data of the measurements. The results are shown in figure 6, as a function of the measurement number h. The parameters L''_q and L_{aQ} behave as expected; apparently R_Q depends heavily upon these parameters. The overall increase in the value of R_a is probably caused by a rise of temperature, but the changes are small compared to the measurement uncertainty (0.1%).

6.3. PREVIOUS MEASUREMENTS

Comparison of the obtained estimation results with the results of previous measurements shows a relatively close similarity. From measurements carried out by Hoeijmakers [10], four parameters are known:

R_a = 0.232 Ω
L''_q = 3.5 mH (at 50 Hz)
R_Q = 0.2 Ω
L_q = 19.3 mH

The quadrature-axis synchronous inductance, L_q, has been measured by applying a positive excitation after first applying a high negative current. Since $L_q = L''_q + L_{aQ}$ it follows that L_{aQ} = 15.8 mH. This value should be compared to the peak values in figure 9d, since the corresponding measurements have also been preceded by armature currents with a sign opposite to the sign of the response current.

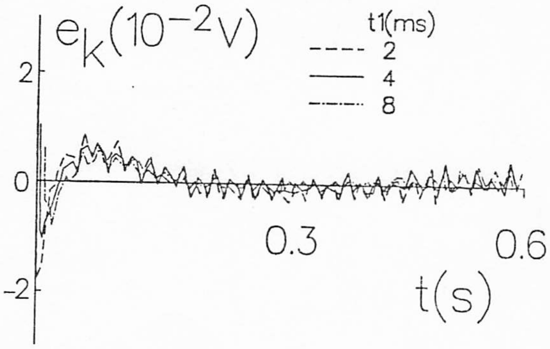

Figure 5

Residuals of some estimations of figure 4

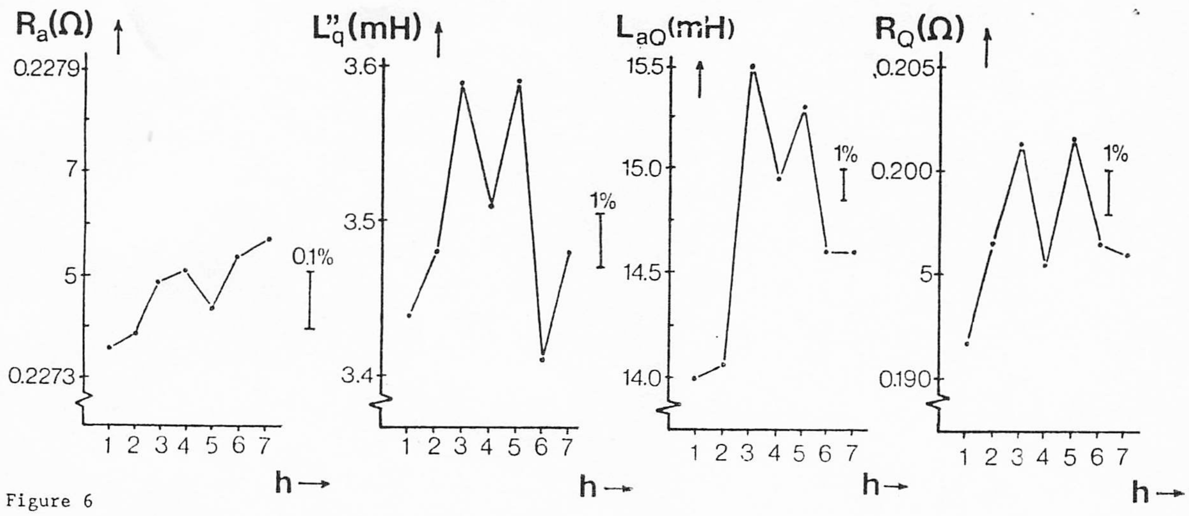

Figure 6

Effects of hysteresis on estimation results

7. CONCLUSIONS

The article presents a method to determine synchronous machine parameters, which is based on a standstill stepresponse test and a least-squares estimator. The machine is excited by an approximate voltage step, which is recorded along with the current response of the machine. The exact characteristics of the excitation source are of minor concern and therefore the measurement set-up can be very simple.

The necessary measurements have been shown to be practical. The thyristor circuit used for the input signal generation is satisfactory and can easily be adapted to accommodate higher power excitations. The computational effort is small enough for a personal computer.

The model implementation is flexible and easy to change. Q-axis parameter estimations carried out at a 24 kVA machine have shown the estimation to proceed satisfactorily.

The disadvantages of the estimation scheme are related to its flexibility. A couple of choices have to be made, which are not immediately obvious: the model-function and more or less accurate starting values of the parameters, which the non-linear minimization algorithm needs for initialization.

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