THREE-PHASE EQUIVALENT CIRCUITS FOR NETWORK SIMULATION OF INDUCTION MACHINES

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Abstract. For the investigation of converters, network simulation programs may be skilful tools. On the other hand, induction machines are mostly described by differential equations. The combination of a converter and an induction machine may also be investigated in a network simulation program. However, in this case the rotor postion angle dependency of the mutual inductances between stator and rotor results in a lot of computation time. In this paper, three-phase equivalent circuits for induction machines (with accessible star connection point) are derived in which the rotor angle dependency is eliminated.
Starting from a general description of an induction machine, the rotor angle dependency is eliminated by transforming (rotating) the rotor variables from the rotor to the stator reference system. Next, the circuit is simplified by referring the rotor quantities to the stator. At the end, this circuit is simplified for the case the star connection point is not used.

Keywords. Induction machine, network simulation

INTRODUCTION

For the investigation of the behaviour of power electronic converters, network simulation programs like Spice may be skilful tools. On the other hand, the description of induction machines is mostly given by differential equations, which may be solved by simulation packages like ACSL and MatrixX. These differential equations are normally derived by using a transformation from three to two phases and next a transformation in order to eliminate the dependency of the mutual inductances on the rotor position angle.
For the simulation of the combination of a converter and an induction machine, the two different kinds of descriptions have to be combined, which may be done in a network simulation program (Arkadan et al (1)). However, in this way the rotor angle dependency results in a lot of computation time (sine and cosine functions). This problem may be avoided by using equivalent circuits for induction machines in which the rotor angle dependency is eliminated (Ronkowski et al (2)).
However, some of the derived equivalent circuits in this paper have the possibility that the star connection point may be used (the homopolar component of the stator currents does not have to be zero), which is in contrast with (2). This may be required when a converter topology is used in which the star connection point is used. Such a topology may be useful for handling fault conditions (Lipo (3)). The knowledge of the star point voltage may also be important for the investigation of the voltage distribution in the machine for insulation problems in case of fault conditions or in case of a current source invertor with an asymmetric dc link (smoothing coil in only one pole of the link).
Besides, in this paper, the finally given circuits have no nodes with more than two inductances. This results into a better numerical behaviour than the circuits given by (2), in which nodes with more than two inductances occur. Furthermore, the finally derived equivalent circuit (for a squirrel cage rotor) in this paper has less nodes than the circuit presented in (2).
The equivalent circuits will be derived starting from a general description of an induction machine with three phases on the stator and three phases on the rotor. The first step is eliminating the rotor angle dependency in the matrix of mutual inductances between stator and rotor by transforming (rotating) the rotor variables from the rotor to the stator reference system. Next, the derived circuit is simplified by referring the rotor quantities to the stator. At the end, this circuit is simplified for the case the star connection point is not used.

THE BASIC EQUATIONS

For the basic description of the induction machine figure 1 will be used.

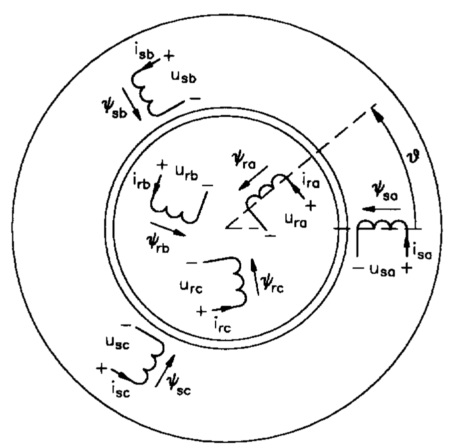

Figure 1 A schematic representation of the induction machine

The stator and the rotor voltage equations are represented by using the vector notation:

$$\vec{u}_{sabc} = R_s \vec{i}_{sabc} + \frac{d\vec{\Psi}_{sabc}}{dt} \;;\; \vec{u}_{rabc} = R_r \vec{i}_{rabc} + \frac{d\vec{\Psi}_{rabc}}{dt} \qquad (1)$$

in which R_r and R_r are the stator and the rotor resistance, respectively, and in which the vectors are given by

$$\vec{u}_{sabc} = \begin{bmatrix} u_{sa} \\ u_{sb} \\ u_{sc} \end{bmatrix} \;;\; \vec{i}_{sabc} = \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} \;;\; \vec{\Psi}_{sabc} = \begin{bmatrix} \Psi_{sa} \\ \Psi_{sb} \\ \Psi_{sc} \end{bmatrix} \;;$$

$$\vec{u}_{rabc} = \begin{bmatrix} u_{ra} \\ u_{rb} \\ u_{rc} \end{bmatrix} \; ; \; \vec{i}_{rabc} = \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} \; ; \; \vec{\Psi}_{rabc} = \begin{bmatrix} \Psi_{ra} \\ \Psi_{rb} \\ \Psi_{rc} \end{bmatrix} \tag{2}$$

The fluxes are expressed by

$$\vec{\Psi}_{sabc} = \left(L_{s\sigma abc}+L_{smabc}\right)\vec{i}_{sabc} + M_{abc}\vec{i}_{rabc} \tag{3a}$$

$$\vec{\Psi}_{rabc} = \mathbf{M}^T_{abc}\vec{i}_{sabc} + \left(L_{r\sigma abc}+L_{rmabc}\right)\vec{i}_{rabc} \tag{3b}$$

in which the matrixes for the main flux $\mathbf{L}_{smabc}$, $\mathbf{L}_{rmabc}$, and $\mathbf{M}_{abc}$ are:

$$L_{smabc} = \begin{bmatrix} \frac{2}{3}L_{sm} & -\frac{1}{3}L_{sm} & -\frac{1}{3}L_{sm} \\ -\frac{1}{3}L_{sm} & \frac{2}{3}L_{sm} & -\frac{1}{3}L_{sm} \\ -\frac{1}{3}L_{sm} & -\frac{1}{3}L_{sm} & \frac{2}{3}L_{sm} \end{bmatrix} \tag{4a}$$

$$L_{rmabc} = \begin{bmatrix} \frac{2}{3}L_{rm} & -\frac{1}{3}L_{rm} & -\frac{1}{3}L_{rm} \\ -\frac{1}{3}L_{rm} & \frac{2}{3}L_{rm} & -\frac{1}{3}L_{rm} \\ -\frac{1}{3}L_{rm} & -\frac{1}{3}L_{rm} & \frac{2}{3}L_{rm} \end{bmatrix} \tag{4b}$$

$$M_{abc} = \begin{bmatrix} \frac{2}{3}\hat{M}\cos(\theta) & \frac{2}{3}\hat{M}\cos(\theta+\frac{2}{3}\pi) & \frac{2}{3}\hat{M}\cos(\theta-\frac{2}{3}\pi) \\ \frac{2}{3}\hat{M}\cos(\theta-\frac{2}{3}\pi) & \frac{2}{3}\hat{M}\cos(\theta) & \frac{2}{3}\hat{M}\cos(\theta+\frac{2}{3}\pi) \\ \frac{2}{3}\hat{M}\cos(\theta+\frac{2}{3}\pi) & \frac{2}{3}\hat{M}\cos(\theta-\frac{2}{3}\pi) & \frac{2}{3}\hat{M}\cos(\theta) \end{bmatrix} \tag{4c}$$

and the matrixes for the leakage flux $\mathbf{L}_{s\sigma abc}$ and $\mathbf{L}_{r\sigma abc}$ are:

$$L_{s\sigma abc} = \begin{bmatrix} L_{s\sigma}+M_{s\sigma} & M_{s\sigma} & M_{s\sigma} \\ M_{s\sigma} & L_{s\sigma}+M_{s\sigma} & M_{s\sigma} \\ M_{s\sigma} & M_{s\sigma} & L_{s\sigma}+M_{s\sigma} \end{bmatrix} \tag{5a}$$

$$L_{r\sigma abc} = \begin{bmatrix} L_{r\sigma}+M_{r\sigma} & M_{r\sigma} & M_{r\sigma} \\ M_{r\sigma} & L_{r\sigma}+M_{r\sigma} & M_{r\sigma} \\ M_{r\sigma} & M_{r\sigma} & L_{r\sigma}+M_{r\sigma} \end{bmatrix} \tag{5b}$$

The matrix elements have been defined in such a way that expressions found further on will have an easy form. The main inductances are supposed to have an ideal coupling, such that

$$\left(\frac{2}{3}L_{sm}\right)\left(\frac{2}{3}L_{rm}\right) = \left(\frac{2}{3}\hat{M}\right)^2 \tag{6}$$

may be used.
The electromagnetic torque follows from:

$$m = \frac{\partial W'_m}{\partial\theta} = \vec{i}^T_{sabc}\,\frac{dM_{abc}}{d\theta}\,\vec{i}_{rabc} \tag{7}$$

For the three-phase description, the matrix

$$C_R = \begin{bmatrix} 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \end{bmatrix} \tag{8}$$

may be useful. Since

$$\frac{dM_{abc}}{d\theta} = -\,C_R M_{abc} \tag{9}$$

the torque may also be expressed as

$$m = -\,\vec{i}^T_{sabc} C_R M_{abc}\vec{i}_{rabc} \tag{10}$$

ROTATION OF ROTOR VARIABLES TO THE STATOR REFERENCE SYSTEM

In order to eliminate the rotor angle dependency in the matrix $\mathbf{M}_{abc}$ ((4c)), the vectors with rotor variables are transformed according to

$$\vec{i}'_{rabc} = \begin{bmatrix} i'_{ra} \\ i'_{rb} \\ i'_{rc} \end{bmatrix} = C_{abc}\vec{i}_{rabc} \; ; \; \vec{u}'_{rabc} = \begin{bmatrix} u'_{ra} \\ u'_{rb} \\ u'_{rc} \end{bmatrix} = C_{abc}\vec{u}_{rabc}$$

$$\vec{\Psi}'_{rabc} = \begin{bmatrix} \Psi'_{ra} \\ \Psi'_{rb} \\ \Psi'_{rc} \end{bmatrix} = C_{abc}\vec{\Psi}_{rabc} \tag{11}$$

where

$$C_{abc} = \begin{bmatrix} \frac{1}{3}+\frac{2}{3}\cos(\theta) & \frac{1}{3}+\frac{2}{3}\cos(\theta+\frac{2}{3}\pi) & \frac{1}{3}+\frac{2}{3}\cos(\theta-\frac{2}{3}\pi) \\ \frac{1}{3}+\frac{2}{3}\cos(\theta-\frac{2}{3}\pi) & \frac{1}{3}+\frac{2}{3}\cos(\theta) & \frac{1}{3}+\frac{2}{3}\cos(\theta+\frac{2}{3}\pi) \\ \frac{1}{3}+\frac{2}{3}\cos(\theta+\frac{2}{3}\pi) & \frac{1}{3}+\frac{2}{3}\cos(\theta-\frac{2}{3}\pi) & \frac{1}{3}+\frac{2}{3}\cos(\theta) \end{bmatrix} \tag{12}$$

The matrix $\mathbf{C}_{abc}$ has the feature:

$$C_{abc}C^T_{abc} = C^T_{abc}C_{abc} = I \tag{13}$$

It should be noted that this transformation is only significant for induction machines with squirrel cage rotors and not for machines with slip rings and that the stator quantities remain the same.
Substitution of (11) with (12) into the rotor voltage equation (1b) and into the flux expressions (3) (with (4) and (5)) results in

$$\vec{u}'_{rabc} = R_r\vec{i}'_{rabc} + C_{abc}\frac{dC^T_{abc}}{dt}\vec{\Psi}'_{rabc} + \frac{d\vec{\Psi}'_{rabc}}{dt} \tag{14a}$$

$$\vec{\Psi}_{sabc} = \left(L_{s\sigma abc}+L_{smabc}\right)\vec{i}_{sabc} + M_{abc}C^T_{abc}\vec{i}'_{rabc} \tag{14b}$$

$$\vec{\Psi}'_{rabc} = C_{abc}M^T_{abc}\vec{i}_{sabc} + C_{abc}\left(L_{r\sigma abc}+L_{rmabc}\right)C^T_{abc}\vec{i}'_{rabc} \tag{14c}$$

Some of the coefficients in these equations may be simplified. Using (8) and (12), the coefficient in the second term in the right part of (14a) becomes

$$C_{abc}\frac{dC^T_{abc}}{dt} = \omega_m C_R \qquad (15a)$$

where

$$\omega_m = \frac{d\theta}{dt} \qquad (15b)$$

Using (4c) and (12), the coefficient in the second term in the right part of (14b) may be defined as

$$M'_{abc} = M_{abc}C^T_{abc} = \begin{bmatrix} \frac{2}{3}\hat{M} & -\frac{1}{3}\hat{M} & -\frac{1}{3}\hat{M} \\ -\frac{1}{3}\hat{M} & \frac{2}{3}\hat{M} & -\frac{1}{3}\hat{M} \\ -\frac{1}{3}\hat{M} & -\frac{1}{3}\hat{M} & \frac{2}{3}\hat{M} \end{bmatrix} \qquad (15c)$$

Observing this equation, it may be concluded that the rotor angle dependency has been eliminated. Furthermore, the coefficient in the second term in the right part of (14c) may be simplified by using (4b), (5b) and (12):

$$C_{abc}\left(L_{r\sigma abc}+L_{rmabc}\right)C^T_{abc} = L_{r\sigma abc} + L_{rmabc} \qquad (15d)$$

Hence, the equations (14) become

$$u'_{rabc} = R_r i'_{rabc} + \omega_m C_R \psi'_{rabc} + \frac{d\psi'_{rabc}}{dt} \qquad (16a)$$

$$\psi_{sabc} = \left(L_{s\sigma abc}+L_{smabc}\right) i_{sabc} + M'_{abc} i'_{rabc} \qquad (16b)$$

$$\psi'_{rabc} = C_{abc}M^T_{abc} i_{sabc} + \left(L_{r\sigma abc}+L_{rmabc}\right) i'_{rabc} \qquad (16c)$$

The torque expression has become

$$\begin{aligned} m &= - i^T_{sabc} C_R M_{abc} i_{rabc} \\ &= - i^T_{sabc} C_R \left(M'_{abc}C_{abc}\right)\left(C^T_{abc} i_{rabc}\right) \\ &= - i^T_{sabc} C_R M'_{abc} i'_{rabc} \end{aligned} \qquad (17)$$

The vector notation will be left now and, for reasons of symmetry, only one phase will be considered. The voltage equations may now be given by:

$$u_{sa} = R_s i_{sa} + \frac{d\psi_{sa}}{dt} \qquad (18a)$$

$$u'_{ra} = R_r i'_{ra} + \frac{d\psi'_{ra}}{dt} + \omega_m \frac{\psi'_{rb}-\psi'_{rc}}{\sqrt{3}} \qquad (18b)$$

where

$$\begin{aligned} \psi_{sa} &= (L_{s\sigma}+L_{sm}) i_{sa} + \left(M_{s\sigma}-\frac{L_{sm}}{3}\right)(i_{sa}+i_{sb}+i_{sc}) \\ &\quad + \hat{M} i'_{ra} - \frac{\hat{M}}{3}(i'_{ra}+i'_{rb}+i'_{rc}) \end{aligned} \qquad (18c)$$

$$\begin{aligned} \psi'_{ra} &= \hat{M} i_{sa} - \frac{\hat{M}}{3}(i_{sa}+i_{sb}+i_{sc}) \\ &\quad + (L_{r\sigma}+L_{rm}) i'_{ra} + \left(M_{r\sigma}-\frac{L_{rm}}{3}\right)(i'_{ra}+i'_{rb}+i'_{rc}) \end{aligned} \qquad (18d)$$

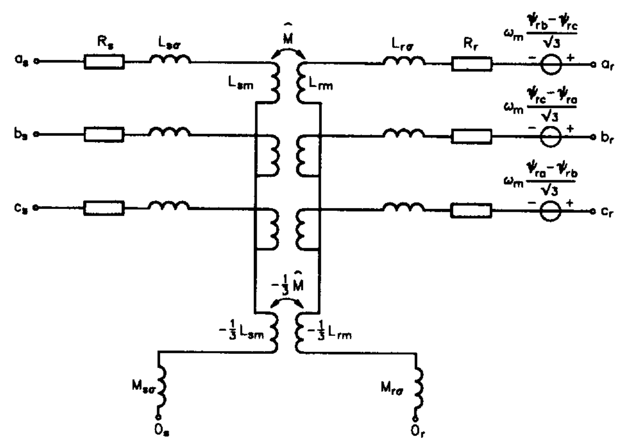

Figure 2 The equivalent circuit for the case in which the rotor variables are transformed to the stator reference system

From these equations figure 2 may be derived. It should be noted that the transformers in this figure are ideally coupled.
The torque expression may now be written as

$$m = \frac{\hat{M}}{\sqrt{3}}\left\{i_{sa}(i'_{rc}-i'_{rb}) + i_{sb}(i'_{ra}-i'_{rc}) + i_{sc}(i'_{rb}-i'_{ra})\right\} \qquad (19)$$

Because of the transformation, the rotor points in figure 2 can not directly be used in a network simulation program.

REFERRING ROTOR QUANTITIES TO THE STATOR

The circuit in figure 2 may be simplified by referring the rotor quantities to the stator. This will be done for squirrel cage rotors, in which the homopolar component of the rotor currents is zero:

$$i'_{ra} + i'_{rb} + i'_{rc} = 0 \qquad (20)$$

The rotor variables will be transformed according to

$$i_{Ra} = \frac{1}{C''_{abc}} i'_{ra} \ ; \ u_{Ra} = C''_{abc} u'_{ra} \ ;$$

$$\psi_{Ra} = C''_{abc} \psi'_{ra} \qquad (21)$$

where

$$C''_{abc} = \frac{\hat{M}}{L_{rm}+L_{r\sigma}} \qquad (22)$$

It should be noted that rotor quantities referred to the stator have the subscript (capital) R. Substitution of (21) with (20) and (22) into the voltage and flux equations (18) gives:

$$u_{sa} = R_s i_{sa} + \frac{d\psi_{sa}}{dt} \qquad (23a)$$

$$u_{Ra} = C''^2_{abc} R_r i_{Rr} + \frac{d\psi_{Ra}}{dt} + \omega_m \frac{\psi_{Rb}-\psi_{Rc}}{\sqrt{3}} \qquad (23b)$$

$$\psi_{sa} = (L_{s\sigma}+L_{sm}) i_{sa} + \left(M_{s\sigma}-\frac{L_{sm}}{3}\right)(i_{sa}+i_{sb}+i_{sc}) + C''_{abc}\hat{M} i'_{Ra} \qquad (24a)$$

$$\psi_{Ra} = C^{//}_{abc}\hat{M} i_{sa} - C^{//}_{abc}\frac{\hat{M}}{3}(i_{sa}+i_{sb}+i_{sc}) + C^{//2}_{abc}(L_{r\sigma}+L_{rm}) i_{Ra} \tag{24b}$$

After introducing

$$L_s = L_{s\sigma} + L_{sm} \tag{25a}$$

$$\sigma = 1 - \frac{\hat{M}^2}{(L_{sm}+L_{s\sigma})(L_{rm}+L_{r\sigma})} \tag{25b}$$

$$R_R = \left[\frac{\hat{M}}{L_{rm}+L_{r\sigma}}\right]^2 R_r \tag{25c}$$

these equations become:

$$u_{sa} = R_s i_{sa} + \frac{d\psi_{sa}}{dt} \tag{26a}$$

$$u_{Ra} = R_R i_{Ra} + \frac{d\psi_{Ra}}{dt} + \omega_m \frac{\psi_{Rb}-\psi_{Rc}}{\sqrt{3}} \tag{26b}$$

$$\psi_{sa} = \sigma L_s i_{sa} + (1-\sigma)L_s(i_{sa}+i_{Ra}) + \left(M_{s\sigma} - \frac{L_{sm}}{3}\right)(i_{sa}+i_{sb}+i_{sc}) \tag{27a}$$

$$\psi_{Ra} = (1-\sigma)L_s(i_{sa}+i_{Ra}) - \frac{(1-\sigma)L_s}{3}(i_{sa}+i_{sb}+i_{sc}) \tag{27b}$$

Using these equations figure 3 may be derived.
The torque expression becomes:

$$m = -i_{sa}\frac{\psi_{rb}-\psi_{rc}}{\sqrt{3}} - i_{sb}\frac{\psi_{rc}-\psi_{ra}}{\sqrt{3}} - i_{sc}\frac{\psi_{ra}-\psi_{rb}}{\sqrt{3}} \tag{28}$$

Since the rotor windings of a squirrel cage rotor are short circuited, the rotor phase voltages are equal:

$$u_{Ra} = u_{Rb} = u_{Rc} \tag{29}$$

Using (20), this results in:

$$u_{Ra} = u_{Rb} = u_{Rc} = 0 \tag{30}$$

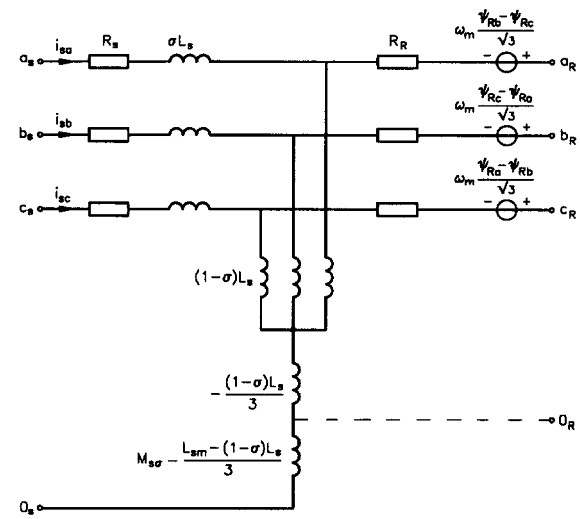

Figure 3 The equivalent circuit after referring the rotor quantities to the stator and omitting the homopolar rotor current component

Hence, the points a_R, b_R, c_R, and 0_R in figure 3 may be connected together for the equivalent circuit of a squirrel cage machine. When a slipring machine is used, the rotor star point may only be used for measurements, as indicated by the dashed line. However, the stator star point 0_s may still be connected to the converter.

NOT USING THE STATOR STAR POINT

When the stator star point is not used, which is so in most cases, the homopolar component of the stator currents equals zero:

$$i_{sa} + i_{sb} + i_{sc} = 0 \tag{31}$$

In this case the voltage equations (26) are still valid. The flux expression (27) become

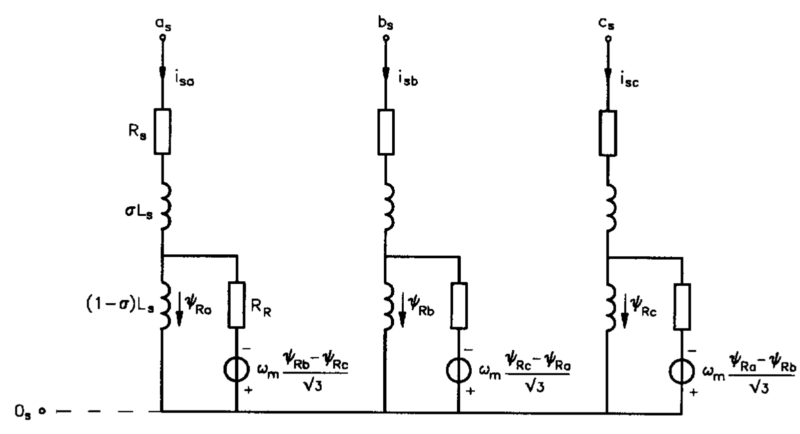

Figure 4 The equivalent circuit for the case the stator star point is not connected

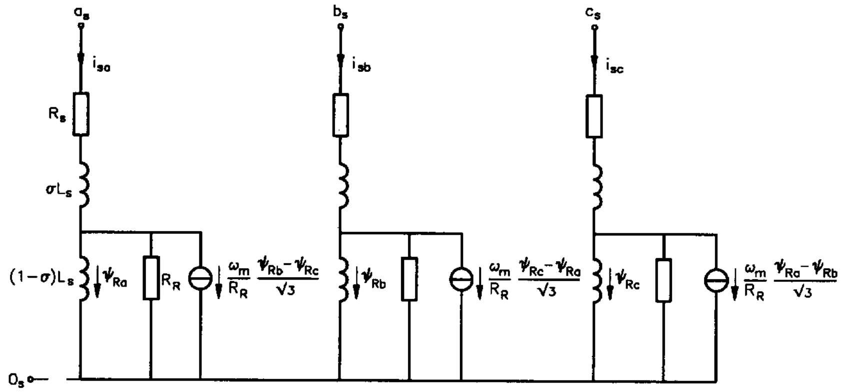

Figure 5 The result of replacing the series connection of a voltage source and a resistor by the parallel connection of a current source and the same resistor

$$\psi_{sa} = \sigma L_s i_{sa} + (1-\sigma)L_s(i_{sa}+i_{Ra}) \tag{32a}$$

$$\psi_{Ra} = (1-\sigma)L_s(i_{sa}+i_{Ra}) \tag{32b}$$

The resulting equivalent circuit is represented in figure 4.
Although the stator star point is not used, this circuit may still be used to determine the floating star point voltage as indicated by the dashed line in figure 4.

A POSSIBLE EXTRA SIMPLIFICATION

When the network simulation package uses the MNA method (Ho and Ruehli (4)) to define the network equations, it may be useful to eleminate nodes in the circuit. In the circuit of figure 4 this is done by replacing the series connection of a voltage source and a resistor by the parallel connection of a current source and the same resistor. This is illustrated in figure 5. It should be noted that this simplification method may also be used for other circuits (figures 2 and 3).

An example

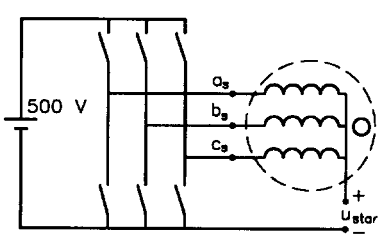

Figure 6 The induction machine fed by a voltage source inverter

The model shown in figure 5 connected to a voltage source inverter as displayed in figure 6 was simulated. In this simulation the stator point voltage is observed but not connected to the inverter circuit. Block modulation is applied to the induction machine, while the rotor angular speed is constant. The parameters used in this example are R_s=0.23Ω, L_s=76.7mH, σ=0.073, ω_m=100rad/s and R_R=0.23Ω. Figure 7 displays the start of the converter while the induction machine is rotating at rated speed. The upper window shows the voltage u_{as}-u_{bs}; the middle window shows the current i_{as} and the observed voltage u_{star} on the stator point 0_s is displayed in the lower window. A fault condition is invoked by changing the value of σL_s for phase *a* to $\sigma L_s/2$ (internal short circuit), which results in a stator star point voltage as displayed in figure 8.

CONCLUSION

The equivalent circuits for an induction machine presented in this paper may be applied for network analysis where the combination of a converter and induction machine has to be studied. Some of the derived equivalent circuits have the possibility that the star connection point may be used.
The derived circuits are presented by lumped circuit elements and the circuit for the squirrel cage rotor is very small and simple to implement compared to circuits known from literature. In this circuit the stator star point may not be used, but its voltage can be observed in the circuit, which makes the circuit suitable to investigate fault conditions.
Because of the small amount of components and because non-linear components have been avoided (they have been eliminated by the transformation of the rotor variables to the stator reference system) the model can easily be applied in network analysis programs.

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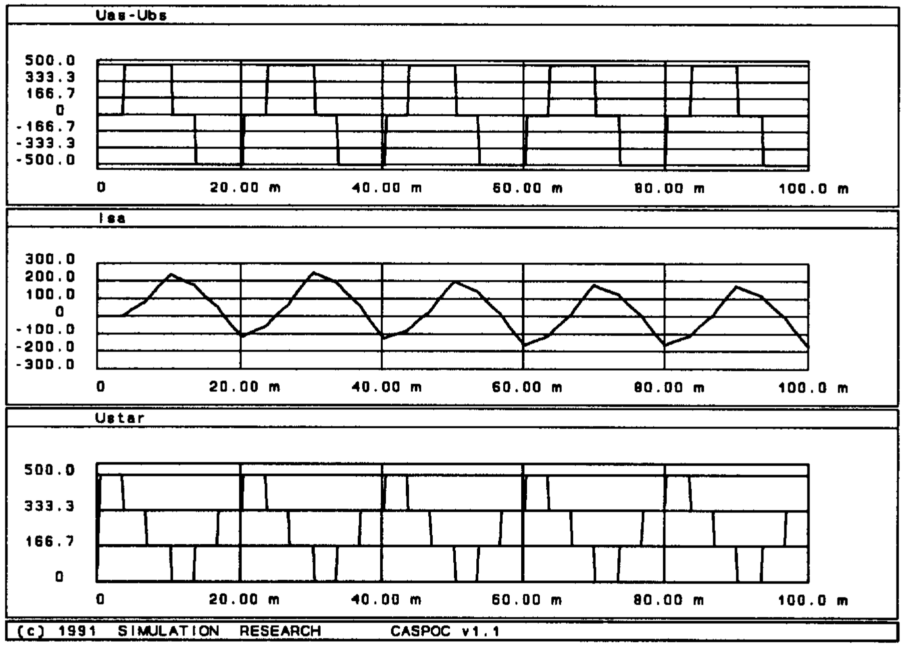

Figure 7 Some simulation results for the circuit shown in figure 6

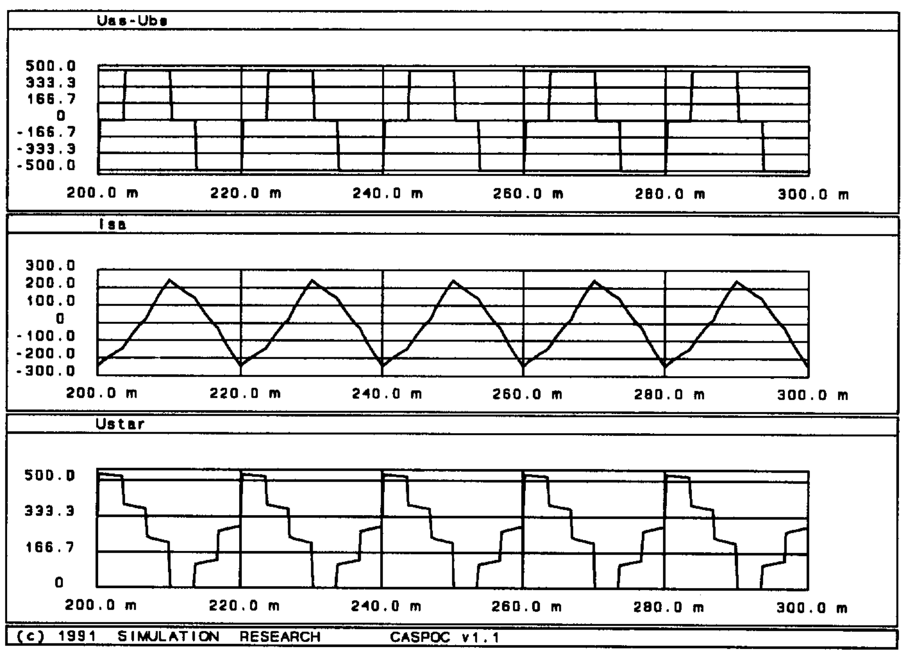

Figure 8 Simulation of a fault in phase a in the circuit shown in figure 6