A simple model of a synchronous machine with rectifier

M.J. Hoeijmakers, Delft University of Technology
Faculty of Electrical Engineering, Power Electronics & Electrical Machines Group
Mekelweg 4, 2628 CD Delft, The Netherlands
Facsimile: + 31 15 278 2968, E-mail: M.J.Hoeijmakers@ET.TUDelft.NL

Abstract

In order to investigate the dynamic behaviour of the synchronous machine with (controllable) rectifier, a detailed simulation is used in most cases, which requires a lot of computation time. Here, a more superficial simulation method is presented.
In this case the subtransient inductances are (imaginarily) splitted off from the machine. The harmonics on the stator voltages of the remaining (imaginary) machine are neglected, so that these voltages may be supposed to be sinusoidal and may be seen as the voltage sources for the rectifier with the subtransient inductances as commutation inductances. Furthermore, the ripple on the direct current is neglected, so that the classic model of a three-phase bridge rectifier (including a global modeling of commutation) may be used. On the ac side of the bridge only the basic harmonics of the currents are considered. Using these assumptions, a set of equations for the synchronous machine with rectifier is derived.
Finally, some attention is paid to the experimental verification of the model.

Keywords

synchronous machine, rectifier, wind energy

1 Introduction

Wind energy

The importance of wind energy for electricity generation is slowly growing. As a result, more attention is paid to the drive system of the wind-energy conversion system, in which energy is converted from mechanical energy into electrical energy (the power flow is negative). The drive system may operate at a (nearly) constant or at a variable speed.
On the average, a variable-speed system will be more complex and more expensive than a constant-speed system. On the other hand, the important advantages of the variable-speed system are that the wind turbine can be optimally loaded, that torque shocks (particularly those resulting from disturbances of the utility-grid) in the mechanical transmission can be limited by a good torque control (cheaper and lighter transmission, and yet a long life) and that it is possible to store kinetic energy in the rotor, which reduces short-term fluctuations (resulting from wind-speed variations) in the shaft torque and in the electrical power. Moreover, in some places it can be important to make the wind turbine work at a reduced speed at night, in view of noise pollution.
Once the choice has been made for a variable-speed system, the choice of systems still remains large. A favourite system is a synchronous machine with dc link. This electromechanical conversion system is a cascade connection of a synchronous machine, a rectifier, a choke and an invertor, as indicated in figure 1.
An important advantage of this system compared to other variable-speed systems is its very high efficiency. Furthermore, synchronous machines are usually made brushless.

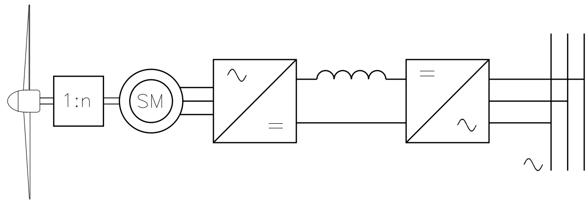

Figure 1 The synchronous machine with dc link in a wind-energy conversion system

Since brush-slipring combinations require relatively much maintenance and since their functioning largely depends on atmospheric circumstances, this is an important fact for the choice of a variable-speed system for wind turbines.

Modelling the synchronous machine with rectifier

The sometimes problematic transient behaviour of the synchronous machine with dc link might be a disadvantage [1]. To investigate the dynamic behaviour of this system, a detailed simulation of the system is mostly used. Such a simulation, in which for example the commutation in the rectifier can be recognized, requires much computation time and is not very useful for control purposes. In this paper a more superficial simulation method is presented.
Here, the subtransient inductances are (imaginarily) splitted off from the synchronous machine. The harmonics on the stator voltages of the remaining (imaginary) machine are neglected, so that these voltages may be supposed to be sinusoidal and may be seen as the voltage sources for the rectifier with the subtransient inductances as commutation inductances. Furthermore, the ripple on the direct current is neglected, so that the classic model of a three-phase bridge rectifier (a model of commutation is included) may be used. On the ac side of the bridge only the basic harmonics of the currents are considered as the phase currents of the synchronous machine. As a result, there is no ripple on the direct-axis and on the quadrature-axis current of the synchronous machine.
The basic ideas of the simple model of the synchronous machine with rectifier based on these assumptions have been published before [2] for the case of a network representation of the machine.
Here, the synchronous machine is modelled by means of its operational inductances (impedances), so that conventional time constants for the direct and for the quadrature axis may be used.
Besides, the model is extended by taking the armature resistance into account. This leads to an implicit set of equations (or algebraic loops), which is not a problem for a simulation program like MATLAB/SIMULINK.
Next, an explicit form of the differential equations is derived by omitting some resistance terms in the voltage equations, which is allowed for a large machine. In this way, a relatively simple set of equations for the synchronous machine with rectifier is found. This set is written as state space

equations, so that they may be used in a normal computer simulation program.
Finally, some attention is paid to the experimental verification of the model.

2 The basic idea of the coupling of the synchronous machine model and the rectifier model

Steady-state operation

Before turning to the rectifier model and the synchronous machine model separately, the basics of the way of coupling those models is explained here.
Only for this explanation the machine is supposed to have only one damper winding on the direct axis and only one damper winding on the quadrature axis. Furthermore, it is supposed that the quadrature-axis and the direct-axis synchronous inductances are equal (L_q=L_d=L_s) and that the quadrature-axis and the direct-axis subtransient inductances are equal (L_q''=L_d''=L''). Later, these assumptions will be set free. For the explanation figure 2 will be used.
The (two-pole) synchronous machine is supposed to be connected to a rectifier which produces a symmetrical three-phase system of currents during steady-state operation.
These currents may be expressed as Fourier series in which all even harmonics are zero thanks to the property $i(\omega t\text{-}\pi)$=$i(\omega t)$, where ω represents the angular speed of the rotor. Moreover, as the star connection terminal of the machine is not used, the armature phase currents do not contain harmonics with an angular frequency which is a multiple of 3ω. Hence, the Fourier series consists of a fundamental component with angular frequency ω and harmonics with angular frequencies of (6*k*-1)ω and (6*k*+1)ω, where *k* is an integer larger than 0.
Since the rotor "sees" the fundamental components of the phase currents as direct currents, they don't induce currents in the rotor circuits. Hence, the impedance of the stator for the fundamental components is determined by the synchronous inductances. Seen from the rotor, the harmonics in the armature currents are transformed into currents with angular frequencies of 6*k*ω. So, these harmonic currents induce currents in the rotor circuits. Because these currents have a relatively high angular frequency, the harmonics in the armature currents see the subtransient inductance as stator inductance.

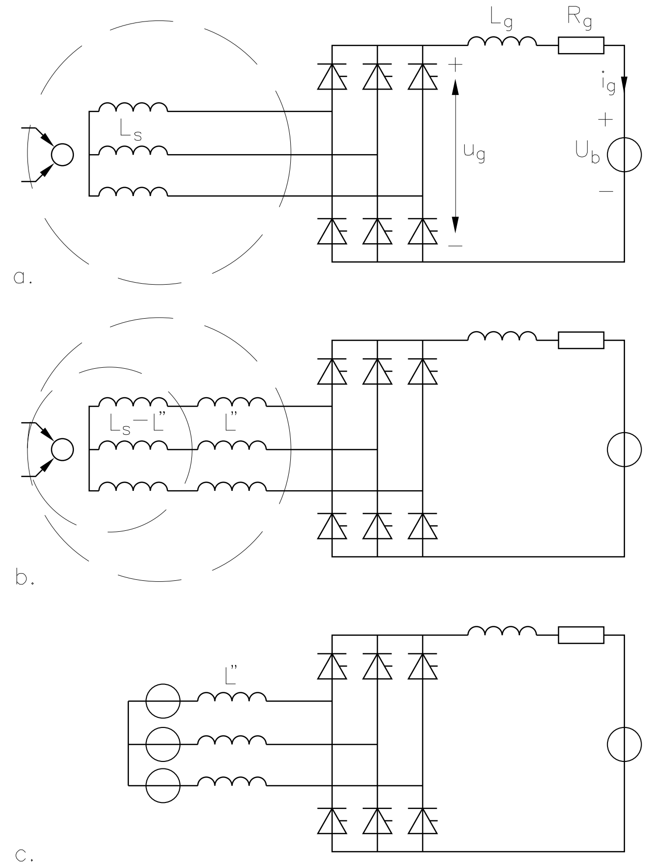

Figure 2 Splitting off the subtransient inductance

In figure 2a the original synchronous machine with synchronous inductance L_s is shown. In figure 2b the subtransient inductance L'' is splitted off by subtracting it from the synchronous inductance. The inner circle of figure 2b represents the so-called internal machine: the original machine minus the subtransient inductances.
Since the phase-current harmonics only see the subtransient inductance, the internal machine (figure 2b) is a short circuit for these harmonics. Hence, the armature voltages of the internal machine are sinusoidal and may be represented by a three-phase voltage source (figure 2c). This voltage source is controlled by the excitation current and the fundamental components of the armature phase currents. So, the voltages of this source do not depend on the higher harmonic currents.
Splitting off the subtransient inductances effectively separates the fast and the slow modes of the machine, thus offering a possibility to simulate machines in this configuration without considering the switching actions in the power electronic rectifier in detail. On the other hand, the three-phase voltage source and the subtransient inductances determine the behaviour of the electronic switches in the rectifier.

Dynamic operation

The idea of separating the fast and the slow modes of the machine may often be used during dynamic operation, because the changes of the amplitude and the frequency of the fundamental components of the armature phase currents are mostly slow compared with the frequencies of the harmonics. These fundamental components with relatively slowly changing amplitude and frequency cause relatively slow transient currents in the damper windings and in the excitation winding on the rotor. These currents have to be taken into account when determining the voltages of the three-phase voltage source in figure 2c. For this purpose, a model of the synchronous machine with splitted off subtransient inductances may be very useful.
In the considered cases, the three-phase voltage source (with relatively slowly changing amplitude and frequency) and the subtransient inductances still determine the behaviour of the electronic switches in the rectifier. Hence, splitting off the subtransient inductances may still be advantageous for investigating the rectifier behaviour.
Based on the previous considerations, a simple model of a synchronous machine has earlier been derived [2].

3 The three-phase bridge rectifier

For the rectifier a well-known model is used, which is explained in more details in, for example, [2]. The derivation of this model is summarized in this section. The base circuit for the model is given in figure 3.
When the ripple on the direct current i_g in this circuit is

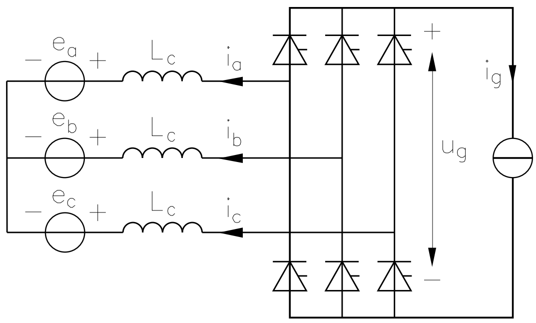

Figure 3 Base circuit for the convertor description

neglected, the commutation angle μ is smaller than π/3, and the phase voltages satisfy

$$\begin{aligned} e_a &= \hat{e}\cos(\omega t+\varepsilon) \\ e_b &= \hat{e}\cos(\omega t-\tfrac{2}{3}\pi+\varepsilon) \\ e_c &= \hat{e}\cos(\omega t-\tfrac{4}{3}\pi+\varepsilon) \end{aligned} \tag{1}$$

the fundamental components of the phase currents may be expressed as:

$$\begin{aligned} i_{a1}(\omega t) &= -\,i_{act}\cos(\omega t+\varepsilon) - i_{rea}\sin(\omega t+\varepsilon) \\ i_{b1}(\omega t) &= -\,i_{act}\cos(\omega t-\tfrac{2}{3}\pi+\varepsilon) - i_{rea}\sin(\omega t-\tfrac{2}{3}\pi+\varepsilon) \\ i_{c1}(\omega t) &= -\,i_{act}\cos(\omega t-\tfrac{4}{3}\pi+\varepsilon) - i_{rea}\sin(\omega t-\tfrac{4}{3}\pi+\varepsilon) \end{aligned} \tag{2}$$

The still arbitrary phase angle ε has been introduced for later use. The active and the reactive component coefficients in the expressions for the currents are given by

$$\begin{aligned} i_{act} &= \frac{\sqrt{3}}{\pi} I_g \{\cos\alpha + \cos(\alpha+\mu)\} \\ i_{rea} &= \frac{3\hat{e}}{2\omega L_c \pi}\{\mu - \sin\mu\cos(2\alpha+\mu)\} \end{aligned} \tag{3}$$

In these equations, α is the delay angle, which equals zero when the rectifier consists of diodes (α=0).
In order to solve the rectifier equations, the equation

$$\cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_c I_g}{\sqrt{3}\hat{e}} \tag{4}$$

is also needed.
The average value of the voltage u_g can be found by

$$U_g = \frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - \frac{3}{\pi}\omega L_c I_g \tag{5}$$

The rectifier description given here may also be used for slow changes in the amplitude or the frequency of the phase voltages and the average value of the direct current. The dynamic model introduced in this way may be improved by enlarging the inductance in the dc circuit with $2L_c$ [3]. Using (5), the equivalent circuit given in figure 4 may be composed.

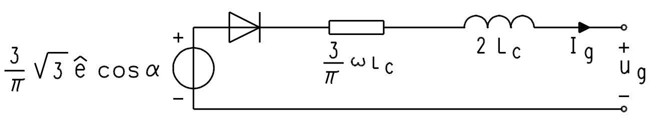

Figure 4 An equivalent circuit for the convertor

4 The synchronous machine

In this section a model of the synchronous machine will be derived. For this model, the usual suppositions are used, see, for example, [4]. The starting point is a description of the machine by means of its operational inductances (impedances). Since it is the intention to make a model for digital simulation, these operational inductances, which are transfer functions, are transformed into state equations.

The Park transformation and the stator voltage equations

In order to describe the machine, the Park transformation according to

$$\begin{aligned} i_d &= \sqrt{\tfrac{2}{3}}\left\{i_a\cos\gamma + i_b\cos(\gamma-\tfrac{2}{3}\pi) + i_c\cos(\gamma-\tfrac{4}{3}\pi)\right\} \\ i_q &= \sqrt{\tfrac{2}{3}}\left\{i_a\sin\gamma + i_b\sin(\gamma-\tfrac{2}{3}\pi) + i_c\sin(\gamma-\tfrac{4}{3}\pi)\right\} \\ i_0 &= \sqrt{\tfrac{1}{3}}\left\{i_a + i_b + i_c\right\} \end{aligned} \tag{6}$$

will be used. The angle γ represents the rotor position. For the phase voltages and flux linkages similar formulas will be used. As may be seen in figure 2, the homopolar current is zero. For that reason no attention is paid to this component. The stator voltage equations are given (ω=dγ/dt) in the usual form:

$$u_d = R_s i_d + \frac{\mathrm{d}\psi_d}{\mathrm{d}t} - \omega\psi_q \quad ; \quad u_q = R_s i_q + \frac{\mathrm{d}\psi_q}{\mathrm{d}t} + \omega\psi_d \tag{7}$$

The direct axis

As has been shown in [5] and in [6], the mostly used model of the synchronous machine with one damper winding is not adequate for the machine which was available for our experimental verification. For that reason, the direct axis is supposed to have two damper windings. This means that the operational inductances (operational impedances) obey

$$\Psi_d = \frac{(1+s\tau_d^{\prime})(1+s\tau_d^{\prime\prime})(1+s\tau_d^{\prime\prime\prime})L_d I_d + (1+s\tau_f^{\prime})(1+s\tau_f^{\prime\prime})\frac{M_{sf}}{R_f}U_f}{(1+s\tau_{d0}^{\prime})(1+s\tau_{d0}^{\prime\prime})(1+s\tau_{d0}^{\prime\prime\prime})}$$

To find the corresponding state equations (time domain), we first expand the transfer function into partial fractions:

$$\Psi_d = L_d^{\prime\prime\prime} I_d + \tau_{d0}^{\prime}\frac{R_d^{\prime} I_d + C_f^{\prime} U_f}{1+s\tau_{d0}^{\prime}} + \tau_{d0}^{\prime\prime}\frac{R_d^{\prime\prime} I_d + C_f^{\prime\prime} U_f}{1+s\tau_{d0}^{\prime\prime}} + \tau_{d0}^{\prime\prime\prime}\frac{R_d^{\prime\prime\prime} I_d + C_f^{\prime\prime\prime} U_f}{1+s\tau_{d0}^{\prime\prime\prime}} \tag{8}$$

Here the coefficients $L_d^{\prime\prime\prime}$, $R_d^{\prime}$, $R_d^{\prime\prime}$, $R_d^{\prime\prime\prime}$, $C_f^{\prime}$, $C_f^{\prime\prime}$, and $C_f^{\prime\prime\prime}$ have been introduced, which follow directly from expanding the transfer function.
The coefficient $L_d^{\prime\prime\prime}$ is the subsubtransient inductance, which is given by

$$L_d^{\prime\prime\prime} = \frac{\tau_d^{\prime}\tau_d^{\prime\prime}\tau_d^{\prime\prime\prime}}{\tau_{d0}^{\prime}\tau_{d0}^{\prime\prime}\tau_{d0}^{\prime\prime\prime}} L_d$$

This inductance is seen when the direct-axis current is changing very rapidly (the Laplace variable s is very large). Now, we introduce the fluxes:

$$\Psi_d^{/} = \tau_{d0}^{/}\frac{R_d^{/}I_d + C_f^{/}U_f}{1+s\tau_{d0}^{/}} \quad ; \quad \Psi_d^{//} = \tau_{d0}^{//}\frac{R_d^{//}I_d + C_f^{//}U_f}{1+s\tau_{d0}^{//}}$$

$$\Psi_d^{///} = \tau_{d0}^{///}\frac{R_d^{///}I_d + C_f^{///}U_f}{1+s\tau_{d0}^{///}} \tag{9}$$

Using these expressions, the Laplace transform of the direct-axis flux Ψ_d according to (8) may be written as

$$\Psi_d = L_d^{///}I_d + \Psi_d^{/} + \Psi_d^{//} + \Psi_d^{///} \tag{10}$$

The equations (9) and (10) are transformed to the time domain:

$$\frac{\mathrm{d}\psi_d^{/}}{\mathrm{d}t} = -\frac{\psi_d^{/}}{\tau_{d0}^{/}} + R_d^{/}i_d + C_f^{/}u_f$$

$$\frac{\mathrm{d}\psi_d^{//}}{\mathrm{d}t} = -\frac{\psi_d^{//}}{\tau_{d0}^{//}} + R_d^{//}i_d + C_f^{//}u_f \tag{11}$$

$$\frac{\mathrm{d}\psi_d^{///}}{\mathrm{d}t} = -\frac{\psi_d^{///}}{\tau_{d0}^{///}} + R_d^{///}i_d + C_f^{///}u_f$$

$$\psi_d = L_d^{///}i_d + \psi_d^{/} + \psi_d^{//} + \psi_d^{///} \tag{12}$$

As we may see, equation (9) passes into the set of state equations (11), which may directly be used in a simulation program.

As an intermediate step to find the direct axis voltage equation, we substitute (11) into the time derivative of the expression for ψ_d (12):

$$\frac{\mathrm{d}\psi_d}{\mathrm{d}t} = L_d^{///}\frac{\mathrm{d}i_d}{\mathrm{d}t} - \frac{\psi_d^{/}}{\tau_{d0}^{/}} - \frac{\psi_d^{//}}{\tau_{d0}^{//}} - \frac{\psi_d^{///}}{\tau_{d0}^{///}} + \left(R_d^{/}+R_d^{//}+R_d^{///}\right)i_d + \left(C_f^{/}+C_f^{//}+C_f^{///}\right)u_f \tag{13}$$

Besides we introduce the coefficients

$$R_d = R_s + R_d^{/} + R_d^{//} + R_d^{///} \quad ; \quad C_f = C_f^{/} + C_f^{//} + C_f^{///}$$

to make the equations more convenient. When we now substitute expression (13) in the direct-axis voltage equation of (7), we find the voltage equation:

$$u_d = R_d i_d - \omega\,\psi_q + L_d^{///}\frac{\mathrm{d}i_d}{\mathrm{d}t} - \frac{\psi_d^{/}}{\tau_{d0}^{/}} - \frac{\psi_d^{//}}{\tau_{d0}^{//}} - \frac{\psi_d^{///}}{\tau_{d0}^{///}} + C_f u_f \tag{14}$$

The quadrature axis

The quadrature axis is supposed to have one damper winding. This means that the operational inductance obeys the mostly used form

$$\Psi_q = \frac{1+s\tau_q^{//}}{1+s\tau_{q0}^{//}}L_q I_q$$

By expanding the transfer function into partial fractions, we find

$$\Psi_q = L_q^{//}I_q + \tau_{q0}^{//}\frac{R_q^{//}I_q}{1+s\tau_{q0}^{//}} \tag{15}$$

Here the coefficients $L_q^{//}$ and $R_q^{//}$ have been introduced, which follow directly from expanding the transfer function. The coefficient $L_q^{//}$ is the subtransient inductance, which is given by

$$L_q^{//} = \frac{\tau_q^{//}}{\tau_{q0}^{//}}L_q$$

This inductance is seen when the quadrature-axis current is changing very rapidly.

After introducing the flux

$$\Psi_q^{//} = \tau_{q0}^{//}\frac{R_q^{//}I_q}{1+s\tau_{q0}^{//}} \tag{16}$$

the Laplace transform of the direct-axis flux Ψ_q according to (15) may be written as

$$\Psi_q = L_q^{//}I_q + \Psi_q^{//} \tag{17}$$

The equations (16) and (17) are transformed to the time domain:

$$\frac{\mathrm{d}\psi_q^{//}}{\mathrm{d}t} = -\frac{\psi_q^{//}}{\tau_{q0}^{//}} + R_q^{//}i_q \tag{18}$$

$$\psi_q = L_q^{//}i_q + \psi_q^{//} \tag{19}$$

Substitution of (18) into the time derivative of the expression for ψ_q (19) gives

$$\frac{\mathrm{d}\psi_q}{\mathrm{d}t} = L_q^{//}\frac{\mathrm{d}i_q}{\mathrm{d}t} - \frac{\psi_q^{//}}{\tau_{q0}^{//}} + R_q^{//}i_q \tag{20}$$

After introducing the coefficient

$$R_q = R_s + R_q^{//}$$

we substitute expression (20) in the quadrature-axis voltage equation of (7):

$$u_q = R_q i_q + \omega\,\psi_d + L_q^{//}\frac{\mathrm{d}i_q}{\mathrm{d}t} - \frac{\psi_q^{//}}{\tau_{q0}^{//}} \tag{21}$$

The set of machine equations

The final step is to eliminate the fluxes ψ_d and ψ_q in the voltage equations (21) and (14) by substituting (12) into (21) and (19) into (14):

$$u_d = R_d i_d - \omega\left(L_q^{//}i_q + \psi_q^{//}\right) + L_d^{///}\frac{\mathrm{d}i_d}{\mathrm{d}t} - \frac{\psi_d^{/}}{\tau_{d0}^{/}} - \frac{\psi_d^{//}}{\tau_{d0}^{//}} - \frac{\psi_d^{///}}{\tau_{d0}^{///}} + C_f u_f$$

$$u_q = R_q i_q + \omega\left(L_d^{///}i_d + \psi_d^{/} + \psi_d^{//} + \psi_d^{///}\right) + L_q^{//}\frac{\mathrm{d}i_q}{\mathrm{d}t} - \frac{\psi_q^{//}}{\tau_{q0}^{//}} \tag{22}$$

The equations (11), (18) and (22) form a set of equations of 6th order and describe the electrical behaviour of the synchronous machine.

5 The coupling of the machine model and the rectifier model

For the coupling of the separate model given in the sections 3 and 4, we start with the adaptation of the machine model to the rectifier model. This takes the largest part of this section. Next, the model of the rectifier circuit and a survey of the complete set of equations are given. Finally some possible further neglects are discussed.

Adaptation of the machine model

If we look at figure 3, we may see that the commutation inductance L_c has to be splitted off in the machine model. For that reason, we introduce the internal voltages as

$$e_a = u_a - L_c\frac{\mathrm{d}i_a}{\mathrm{d}t} \quad ; \quad e_b = u_b - L_c\frac{\mathrm{d}i_b}{\mathrm{d}t} \quad ; \quad e_c = u_c - L_c\frac{\mathrm{d}i_c}{\mathrm{d}t}$$

With the Park transformation according to (6), these equations may be transformed into

$$e_d = u_d - L_c\frac{\mathrm{d}i_d}{\mathrm{d}t} + \omega L_c i_q \quad ; \quad e_q = u_q - L_c\frac{\mathrm{d}i_q}{\mathrm{d}t} - \omega L_c i_d$$

These equations may be worked out further by substituting (22):

$$\begin{aligned} e_d &= R_d i_d - \omega\left(\left(L_q^{//}-L_c\right)i_q + \psi_q^{//}\right) \\ &\quad + \left(L_d^{///}-L_c\right)\frac{\mathrm{d}i_d}{\mathrm{d}t} - \frac{\psi_d^{/}}{\tau_{d0}^{/}} - \frac{\psi_d^{//}}{\tau_{d0}^{//}} - \frac{\psi_d^{///}}{\tau_{d0}^{///}} + C_f u_f \end{aligned} \tag{23}$$

$$\begin{aligned} e_q &= R_q i_q + \omega\left(\left(L_d^{///}-L_c\right)i_d + \psi_d^{/} + \psi_d^{//} + \psi_d^{///}\right) \\ &\quad + \left(L_q^{//}-L_c\right)\frac{\mathrm{d}i_q}{\mathrm{d}t} - \frac{\psi_q^{//}}{\tau_{q0}^{//}} \end{aligned} \tag{24}$$

As has been explained in section 2, the basic idea of the coupling of the models supposes that the Park transformed internal voltages e_d and e_q are relatively slowly changing quantities. This is caused by the supposition that the internal phase voltages e_a, e_b, and e_c are "sinusoidally" changing quantities with relatively slowly changing amplitude and frequency.

The voltages e_d and e_q are made relatively slowly changing by choosing $L_c = L_d^{///}$ in equation (23) and choosing $L_c = L_q^{//}$ in equation (24). In this way the terms with the time derivatives of i_d and i_q, which are very large during commutation in the rectifier, are eliminated. We could expect these choices when we compare figure 2 and figure 3.

However, L_c may only have one value. Choosing the average value appears to be quite adequate [3, 7]:

$$L_c = \frac{L_d^{///} + L_q^{//}}{2} \tag{25}$$

After making this choice, we neglect the terms with the time derivatives of i_d and i_q. Using (25), the equations (23) and (24) may be written as:

$$\begin{aligned} e_d &= R_d i_d - \omega\left(\frac{L_q^{//}-L_d^{///}}{2}i_q + \psi_q^{//}\right) \\ &\quad - \frac{\psi_d^{/}}{\tau_{d0}^{/}} - \frac{\psi_d^{//}}{\tau_{d0}^{//}} - \frac{\psi_d^{///}}{\tau_{d0}^{///}} + C_f u_f \\ e_q &= R_q i_q + \omega\left(\frac{L_d^{///}-L_q^{//}}{2}i_d + \psi_d^{/} + \psi_d^{//} + \psi_d^{///}\right) - \frac{\psi_q^{//}}{\tau_{q0}^{//}} \end{aligned} \tag{26}$$

We hold the voltages e_d and e_q to be relatively slowly changing dc quantities. As mentioned before, such quantities are transformed by the Park transformation to "sinusoidally" changing quantities with relatively slowly changing amplitude and frequency.

Now, we arbitrarily choose the position angle γ to be $-\pi/2$ at t=0 for the Park transformation according to (6):

$$\gamma = \omega t - \tfrac{1}{2}\pi \tag{27}$$

Using (27), the voltages e_d and e_q are transformed into the three-phase set of voltages according to (1). The amplitude ê and the phase ε in this set of voltages are given by

$$\hat{e} = \frac{\sqrt{2}}{\sqrt{3}}\sqrt{e_d^2 + e_q^2} \quad ; \quad \varepsilon = -\arctan\frac{e_d}{e_q} \tag{28}$$

The phase angle ε represents the load angle of the internal machine of figure 2b.

Since only the fundamental components of the phase currents are considered, we may use the set of three-phase currents according to (2). By means of the Park transformation according to (6) and the choice for γ according to (27), this set is transformed into a set of relatively slowly changing dc currents:

$$\begin{aligned} i_d &= -\sqrt{\tfrac{3}{2}}\left\{\hat{i}_{rea}\cos\varepsilon - \hat{i}_{act}\sin\varepsilon\right\} \\ i_q &= -\sqrt{\tfrac{3}{2}}\left\{\hat{i}_{rea}\sin\varepsilon + \hat{i}_{act}\cos\varepsilon\right\} \end{aligned} \tag{29}$$

The model of the rectifier circuit

Figure 4 gives the model of the dc side of the circuit in figure 3. In order to use figure 3 as a model for our system, as depicted in figure 2, we have to add the resistance R_g, the inductance L_g and the voltage source U_b from figure 2 to figure 4. In this way, we find:

$$\frac{\mathrm{d}i_g}{\mathrm{d}t} = \frac{\frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - \left(\frac{3}{\pi}\omega L_c + R_g\right)i_g - U_b}{L_g + 2L_c} \tag{30}$$

The complete set of equations

Now, we have found a complete set of equations for the system in figure 2, which consistst of

- the state equations for the machine fluxes (11) and (18);
- the equations for the internal voltages (26) and (28);
- the state equation for the dc current (30);
- the expression for the commutation inductance (25);
- the equations for the rectifier (3) and (4);
- the phase current equations (29).

This set of equations is an implicit set of equations (with so-called algebraic loops). Such a set may cause problems or may even not be solved by many simulation programs. However, it is not a big problem for SIMULINK/MATLAB.

Some possible further neglects

This set of equations is implicit, because the internal voltages according to (26) do not only depend on state variables, but also on the currents i_d and i_q. However, this dependence may often be neglected.

First, the difference between $L_d^{///}$ and $L_q^{//}$ is often very small, so that the terms with the difference may be neglected. In larger machines, the resistive terms may be neglegible compared to the other terms. Using these neglects, the expressions for the internal voltages (26) become

$$
\begin{aligned}
e_d &= -\,\omega\,\psi_q^{\prime\prime} - \frac{\psi_d^{\prime}}{\tau_{d0}^{\prime}} - \frac{\psi_d^{\prime\prime}}{\tau_{d0}^{\prime\prime}} - \frac{\psi_d^{\prime\prime\prime}}{\tau_{d0}^{\prime\prime\prime}} + C_f u_f \\
e_q &= \omega\left(\psi_d^{\prime} + \psi_d^{\prime\prime} + \psi_d^{\prime\prime\prime}\right) - \frac{\psi_q^{\prime\prime}}{\tau_{q0}^{\prime\prime}}
\end{aligned}
\qquad (31)
$$

As a result, the complete set of equations may be written as an explicit set of state equations of 5th order (the machine is supposed to have two damper windings and an excitation winding on the direct axis and one damper winding on the quadrature axis).

6 The experimental verification

Finally, some measurements on the system of the synchronous machine with a diode rectifier are compared with computation results. More information about the measurement set up may be found in [6].

In the figures 5 and 6, the computed and the measured current in the dc link may be compared. The low-frequency ripple on computed current is caused by an oscillation in the mechanical speed of the system used for the experimental verification.

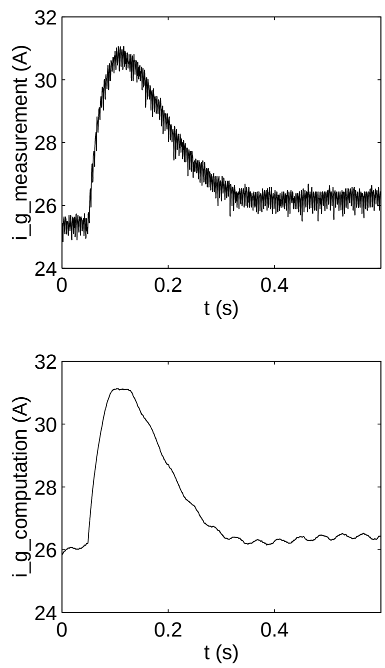

Figure 6 The response on a step on the voltage of the dc link

Conclusion

In this paper a rather simple dynamic model of a synchronous machine with dc-link is presented. For the case where the rectifier consists of diodes and the synchronous machine has two damper windings on the direct axis, the results obtained by means of this simple model have been compared with measurements. It appeared, that using the simple model presented here comes up to the expectations: the short-term averaged values of the system variables are simulated correctly; details, such as the harmonics on the phase currents and the ripple on the direct current are not taken into acccount.

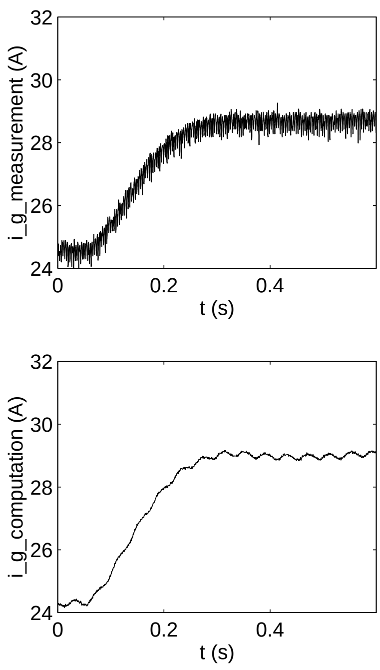

Figure 5 The response on a step on the field voltage

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