Another view on space vectors

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Abstract

Two vectors with a physical meaning for the description of AC machines are introduced. One represents the magnetomotive force; the other represents the fundamental wave of the air-gap flux density. These vectors are linked by means of a simple network representation of the magnetic circuit, in which salient poles may be considered. This network representation also allows for modelling main flux saturation in an easy way.

Keywords

AC machines, space vectors

1 Introduction

For the description of AC machines space phasors or space vectors are often used. Originally, the vector for the current (the magnetomotive force) was the only vector with a physical meaning (Kovacs and Racs, [1]). Later, Stepina improved the physical interpretation for some special cases. A survey of literature on this field is given by him in [2]. Stepina pays special attention to dealing with space harmonics in induction machines.

In this paper, only the fundamental wave is considered. For this case, two vectors with a physical meaning are used. The first one is a vector representing the magnetomotive force, while the second (new) one represents the fundamental wave of the air-gap flux density. Using these vectors, models of AC machines may easily be derived. Especially for the case of salient poles or main flux saturation, the derivation is much more straight forward than conventional derivations.

The essence of the presented method is the use of a simple network representation of the magnetic circuit of the machine (Hopkinson's Law) and the use of symmetry in the magnetic circuit of a salient pole machine. Using the new vector description the conventional machine equations are found, while Clarke components and the transformation from a three-phase machine to a two-phase machine are introduced in a natural way. The Park transformation also arises automatically.

For educational purposes real vectors are used and not complex quantities, because most students have problems with the distinction between space and time phasors. However, this choice is not essential. Besides, real vectors may easily be used in the simulation program MATLAB/-SIMULINK.

In the presented method, first the magnetomotive force is computed from the stator and the rotor currents. Next, the fundamental wave of the air-gap flux is obtained. Then, the (main) flux linked with the windings follows. Finally, the winding voltage equations are found by adding the leakage flux and the winding resistance.

In section 2, the main steps of this process are described for the stator, which is essentially the same for induction and synchronous machines. Next, the basic equations for an induction machine are derived. Finally, the principles of the derivation of the salient-pole synchronous machine equations are given.

2 The stator

In this setion we deal with two steps of the modelling process, namely the computation of the magnetomotive force caused by the stator currents and the relation between the air-gap flux density and the (main) flux linkage of a stator winding. Besides, some expressions for the electromagnetic torque are derived.

An important supposition in the modelling is that the stator winding consists of three sinusoidally distributed windings along the stator circumference in the air gap.

Ampere's Integral Law applied to the stator

We start with observing one stator winding (subscript 1), which is sinusoidally distributed along the stator circumference according to

$$Z_1(\alpha_s) = \hat{Z}_1 \sin(\alpha_s - \alpha_1) \qquad (1)$$

where Z_1 is the number of conductors per metre and α_1 represents the axis of the winding (see figure 1).

Using Faraday's integral law, an expression for the magnetomotive force caused by the current the winding i_1 may be found:

$$F_{m1}(\alpha_s) = i_1 \int_{\alpha_s}^{\alpha_s+\pi} Z_1(\alpha')\, r\, d\alpha' = 2 r i_1 \hat{Z}_1 \cos(\alpha_s - \alpha_1)$$

where r is the stator bore radius. This expression may be simplified by using the total number of turns of the winding, which is found by integrating (1):

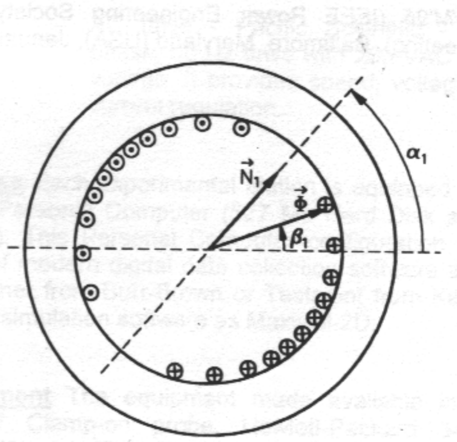

Figure 1 The observed sinusoidally distributed winding

$$N_1 = r\int_{\alpha_1}^{\alpha_1+\pi} Z_1(\alpha')\mathrm{d}\alpha' = 2r\hat{Z}_1 \qquad (2)$$

This results in

$$F_{m,1}(\alpha_s) = i_1 N_1 \cos(\alpha_s - \alpha_1)$$

The observed winding may be represented by the vector $\vec{N}_1$, which is in the direction of the axis of the winding and the length of which corresponds with the total number of turns, as is illustrated in figure 1.

The mmf vector Next, we introduce a vector for the magnetomotive force:

$$\vec{F}_{m,1} = i_1 \vec{N}_1 \qquad (3)$$

The length of this vector equals the maximum value of the magnetomotive force (mmf) and the direction of the vector is the direction of the maximum of the magnetomotive force: the mmf is distributed cosinusoidally around this vector.

The three-phase stator winding Now, the mmf caused by a three-phase stator winding may be found by a vector addition (see figure 2):

$$\vec{F}_{ms} = i_{sa}\vec{N}_{sa} + i_{sb}\vec{N}_{sb} + i_{sc}\vec{N}_{sc}$$

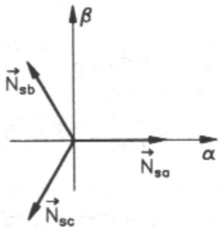

Figure 2 The stator winding vectors

Up to now, the vectors were not represented in a particular coordinate system. When the stator coordinate system is used (with α-axis and β-axis), the vector for the stator mmf is

$$\vec{F}_{ms,\alpha\beta} = i_{sa}N_s\begin{bmatrix}1\\0\end{bmatrix} + i_{sb}N_s\begin{bmatrix}-\frac{1}{2}\\ \frac{1}{2}\sqrt{3}\end{bmatrix} + i_{sc}N_s\begin{bmatrix}-\frac{1}{2}\\ -\frac{1}{2}\sqrt{3}\end{bmatrix}$$
$$= N_s\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2}\sqrt{3}\end{bmatrix}\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{sc}\end{bmatrix} \qquad (4)$$

Faraday's Law applied to the stator

To find an expression for the voltage induced in the winding by means of Faraday's law, we need the flux linked with the winding. In the first instance, the main flux is only considered. Thanks to symmetry properties, an arbitrary flux density distribution in the air gap may be expressed by a Fourier series:

$$B(\alpha_s) = \sum_{m=0}^{\infty} \hat{B}_{2m+1}\cos((2m+1)(\alpha_s - \beta_{2m+1})) \qquad (5)$$

The flux linked with the arbitrary stator winding The next step is to find the flux linked with one turn of the stator winding. The flux linked with a turn with axis $\alpha_s = \alpha'$ is given by (see figure 3):

$$\Phi'(\alpha') = \int_{\alpha'-\frac{\pi}{2}}^{\alpha'+\frac{\pi}{2}} B(\alpha)\, lr\mathrm{d}\alpha$$

where l is the core length.

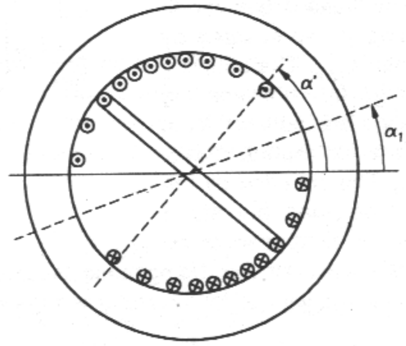

Figure 3 A sinusoidally distributed winding with its magnetic axis and the magnetic axis of a turn

Substituting the fourier series for the magnetic flux density in the air gap $B(\alpha_s)$ according to (5) gives

$$\Phi'(\alpha') = 2lr\sum_{k=0}^{\infty}\frac{\hat{B}_{2k+1}}{2k+1}\sin\left((2k+1)\frac{\pi}{2}\right)\cos((2k+1)(\alpha' - \beta_{2k+1}))$$

Using the expression for the winding distribution (1), we may find for the flux linked with the winding in figure 1 by integration:

$$\psi_1 = \int_{\alpha_1}^{\alpha_1+\pi} Z_1(\alpha' + \frac{\pi}{2})\,\Phi'(\alpha')\, r\mathrm{d}\alpha' = \frac{\pi}{2}lrN_1\hat{B}_1\cos(\beta_1 - \alpha_1) \qquad (6)$$

As we may see, the fundamental component of the air gap flux density is the only component which contributes to the flux linkage of a sinusoidally distributed winding.

The flux vector The fundamental component of the flux density distribution may be represented by the vector $\vec{\Phi}$, which is in the direction of the maximum of the distribution (β_1) and the length of which corresponds with the maximum of the flux density:

$$\Phi = \frac{\pi}{2}lr\hat{B}_1 \qquad (7)$$

The way in which the flux vector may be found will be discussed in the sections 3 and 4. Now, the flux linkage for the winding observed here (figure 1), which was given by (6) may be written as a scalar product:

$$\psi_1 = \vec{\Phi}\cdot\vec{N}_1$$

The three-phase stator winding The fluxes linked with the three stator windings may now be given by:

$$\psi_{sma} = \vec{N}_{sa}\cdot\vec{\Phi} = N_s\begin{bmatrix}1\\0\end{bmatrix}\cdot\begin{bmatrix}\Phi_\alpha\\ \Phi_\beta\end{bmatrix} = N_s\Phi_\alpha$$

$$\psi_{smb} = \vec{N}_{sb}\cdot\vec{\Phi} = N_s\left(-\frac{1}{2}\Phi_\alpha + \frac{\sqrt{3}}{2}\Phi_\beta\right) \qquad (8)$$

$$\psi_{smc} = \vec{N}_{sc}\cdot\vec{\Phi} = N_s\left(-\frac{1}{2}\Phi_\alpha - \frac{\sqrt{3}}{2}\Phi_\beta\right)$$

For the development of these expressions the stator (αβ) coordinate system has been used.

The stator voltage equations and the Clarke transformation

To find the voltage equations for the stator, the leakage flux and the winding resistances have to be allowed for. The leakage flux of one phase winding is represented by the inductance $L_{s\sigma a}$. The coupling by the leakage flux between the phase windings is accounted for in the coefficient for mutual inductance $M_{s\sigma ab}$. The coupling by the leakage flux between the stator and the rotor is supposed to be accounted for in the main flux. Using these assumptions and the expressions (8), we find for the stator voltage equations:

$$\begin{bmatrix} u_{sa} \\ u_{sb} \\ u_{sc} \end{bmatrix} = R_s \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} + \begin{bmatrix} L_{s\sigma a} & M_{s\sigma ab} & M_{s\sigma ab} \\ M_{s\sigma ab} & L_{s\sigma a} & M_{s\sigma ab} \\ M_{s\sigma ab} & M_{s\sigma ab} & L_{s\sigma a} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} + N_s \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \Phi_\alpha \\ \Phi_\beta \end{bmatrix} \tag{9}$$

The voltage equations may be simplified by means of the Clarke transformation. This simplification is based on the fact that the air-gap behaviour of the machine is determined by the two components of the vectors $\vec{\Phi}$ and $\vec{F}_m$, as may be seen in the equations (4) and (9).

Here, we use the normalized Clarke transformation according to

$$\begin{bmatrix} x_{s\alpha} \\ x_{s\beta} \\ x_{s0} \end{bmatrix} = C_{23} \begin{bmatrix} x_{sa} \\ x_{sb} \\ x_{sc} \end{bmatrix} \quad \text{with} \quad C_{23} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix} \tag{10}$$

The basic elements of the transformation matrix automatically arise from the equations (4) and (9).

Using the Clarke transformation, the expression for the mmf vector (4) is simplified to:

$$\vec{F}_{ms\alpha\beta} = \frac{\sqrt{3}}{\sqrt{2}} N_s \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \tag{11}$$

Because in most ac machines the star point connection is not used, the sum of their phase currents is zero. So, there are no zero components. Here, we pay no attention to the zero components. However, when they are of interest, the belonging equations may easily be added.

Using that the zero component of the stator currents is zero, we see in voltage equations (9) that the leakage inductance seen in one phase obeys

$$L_{s\sigma} = L_{s\sigma a} - M_{s\sigma ab}$$

Using the Clarke transformation ((10)), the voltage equations (9) now become:

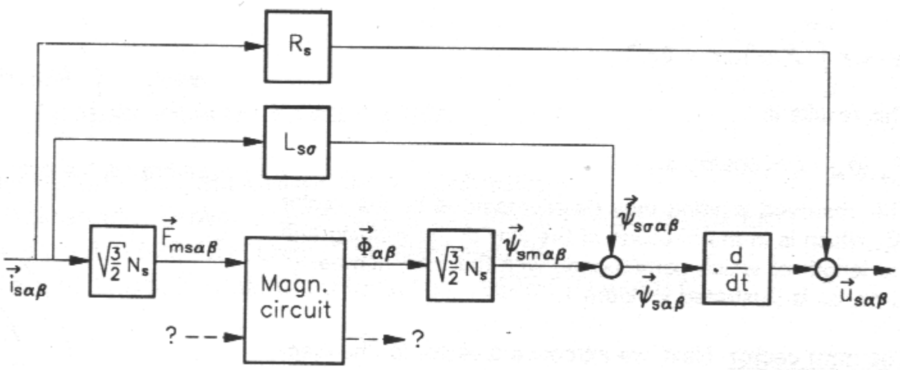

Figure 4 A schematic description of the stator

$$u_{s\alpha} = R_s i_{s\alpha} + L_{s\sigma}\frac{di_{s\alpha}}{dt} + \frac{\sqrt{3}}{\sqrt{2}} N_s \frac{d\Phi_\alpha}{dt}$$

$$u_{s\beta} = R_s i_{s\beta} + L_{s\sigma}\frac{di_{s\beta}}{dt} + \frac{\sqrt{3}}{\sqrt{2}} N_s \frac{d\Phi_\beta}{dt} \tag{12}$$

Next, we also introduce vectors for the voltages, currents, and flux linkages. These vectors do not have any physical (spatial) meaning, what is in contrast with the vectors $\vec{\Phi}$ and $\vec{F}_m$.

The equations (11) and (12) may now be combined to the vector equations

$$\vec{u}_{s\alpha\beta} = R_s \vec{i}_{s\alpha\beta} + L_{s\sigma}\frac{d\vec{i}_{s\alpha\beta}}{dt} + \frac{d\vec{\psi}_{sm\alpha\beta}}{dt}$$

$$\vec{F}_{ms\alpha\beta} = \sqrt{\frac{3}{2}} N_s \vec{i}_{s\alpha\beta} \quad ; \quad \vec{\psi}_{sm\alpha\beta} = \sqrt{\frac{3}{2}} N_s \vec{\Phi}_{\alpha\beta} \tag{13}$$

Using the Clarke transformation, this is a complete set of equations for the three-phase stator winding, which is schematically given in figure 4.

However, it is also a description of a semi-four-phase (or two-phase) stator winding with $\sqrt{3/2}\,N_s$ turns, as depicted in figure 5. This is a physically realizable stator.

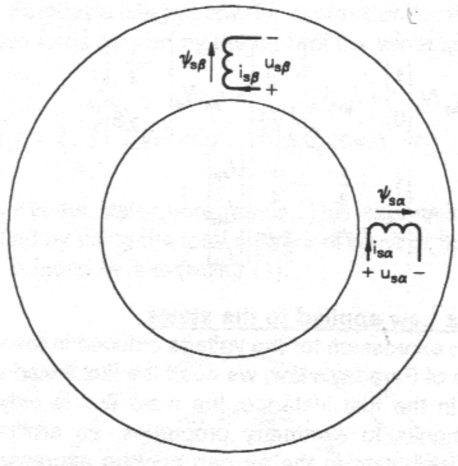

Figure 5 The stator of a two-phase machine

The electromagnetic torque

With the assumed stator winding distribution in the air gap, the electromagnetic torque on the stator originates from the Lorentz force caused by the interaction between stator currents and air-gap flux.

To find an expression for the torque, we start with the tangential Lorentz force on a conductor with (core) length l and current i_1:

$F(\alpha_s) = B(\alpha_s) i_1 l$

This results in the moment of force

$m(\alpha_s) = rB(\alpha_s) i_1 l$

on the stator.
The next step is to find the moment of force on the arbitrary stator winding as depicted in figure 1 by integration:

$$m = rli_1 \int_0^{2\pi} B(\alpha_s)\, Z_1(\alpha_s)\, r\,\mathrm{d}\alpha_s$$

Substituting the expression for the winding distribution (1) and the fourier series for the magnetic flux density in the air gap $B(\alpha_s)$ according to (5) gives

$$m = -\,r^2 l i_1 \hat{Z}_1 \hat{B}_1 \pi \sin(\alpha_1 - \beta_1)$$

From this expression, it becomes clear that only the fundamental component of the magnetic flux density contributes to the moment of force.
If we write this expression as

$$m = -\left(\frac{\pi}{2} rl\hat{B}_1\right)\left(2r\hat{Z}_1\right) i_1 \sin(\alpha_1 - \beta_1)$$

we can easily see that we can also write it as

$$m = -\,\Phi\, N_1\, i_1 \sin(\alpha_1 - \beta_1)$$

by using (2) and (7). This moment of force may be seen as the vector product of $\vec{N}_1$ and $\vec{\Phi}$:

$$\vec{m} = -\,i_1\, \vec{\Phi} \times \vec{N}_1$$

or as the vector product (see (3))

$$\vec{m} = -\,\vec{\Phi} \times \vec{F}_{m1}$$

When we want to have the contribution of all stator windings, we have to use the stator mmf vector. Besides, we mostly want to know the electromagnetic torque on the rotor (instead of the torque on the stator). This torque may be expressed as:

$$\vec{T}_e = \vec{\Phi} \times \vec{F}_{ms}$$

Since this torque vector only has a component in the direction of the shaft, the scalar expression is mostly used:

$$T_e = \Phi_\alpha F_{ms\beta} - \Phi_\beta F_{ms\alpha}$$

We may also write this as a scalar product:

$$T_e = \left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\Phi}\right) \cdot \vec{F}_{ms}$$

With the expression of the mmf vector and the expression for the main flux in (13), this may be worked out to

$$T_e = (-\,i_{s\alpha}\Psi_{sm\beta} + i_{s\beta}\Psi_{sm\alpha}) = \left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\Psi}_{sm\alpha\beta}\right) \cdot \vec{i}_{s\alpha\beta} \qquad (14)$$

3 The induction machine

Here, the induction machine is supposed to have the same kind of windings on the rotor as on the stator.
For the stator, the equations (13) may directly be used. For the rotor, the same equations may be used. However, the subscript s has to be replaced by r and the equations are valid in the rotor coordinate system, which is indicated by the subscripts d and q.

$$\vec{u}_{rdq} = R_r \vec{i}_{rdq} + L_{r\sigma}\frac{\mathrm{d}\vec{i}_{rdq}}{\mathrm{d}t} + \frac{\mathrm{d}\vec{\Psi}_{rmdq}}{\mathrm{d}t} \qquad (15)$$

$$\vec{F}_{mr,dq} = \frac{\sqrt{3}}{\sqrt{2}} N_r \vec{i}_{rdq} \quad ; \quad \vec{\Psi}_{rmdq} = \frac{\sqrt{3}}{\sqrt{2}} N_r \vec{\Phi}_{dq}$$

For the addition of the mmf vectors from (13) and from (15), we have to express them in the same coordinate system, for example in the stator coordinate system. This may be realized by means of the coordinate transformation according to

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = C_{rot}(p\theta) \begin{bmatrix} x_d \\ x_q \end{bmatrix} \quad \text{with} \quad C_{rot}(p\theta) = \begin{bmatrix} \cos p\theta & -\sin p\theta \\ \sin p\theta & \cos p\theta \end{bmatrix} \qquad (16)$$

where p is the number of pole pairs and θ is the rotor position angle.
Using this transformation, the total mmf vector is given by

$$\vec{F}_{m\alpha\beta} = \vec{F}_{ms,\alpha\beta} + \vec{F}_{mr,\alpha\beta} = \sqrt{\tfrac{3}{2}} N_s \vec{i}_{s\alpha\beta} + C_{rot}(p\theta)\sqrt{\tfrac{3}{2}} N_r \vec{i}_{rdq} \qquad (17)$$

Since the rotor of an induction machine is cylindrical, the maximum of the flux density in the air gap is at the same place as the maximum of the mmf. Hence, the mmf vector $\vec{F}_m$ has the same direction as the flux vector $\vec{\Phi}$. The relation between the vectors may be expressed as:

$$\vec{\Phi} = \frac{\vec{F}_m}{R_m} \qquad (18)$$

This expression is a special case of Hopkinson's law. In the reluctance R_m the main flux saturation may directly be incorporated. It should be noted that R_m only depends on the magnetic circuit of the machine and not on the numbers of turns of the windings.
Using the equations (13), (15), (16), (17), and (18) all well-known sets of machine equations may be derived easily. These equations are combined to the schematic description of the induction machine in figure 6.
After defining the inductance coefficients

$$L_{sm} = \frac{\frac{3}{2}N_s^2}{R_m} \quad ; \quad L_{rm} = \frac{\frac{3}{2}N_r^2}{R_m} \quad ; \quad M = \frac{\frac{3}{2}N_s N_r}{R_m}$$

for the main flux and the self-inductance coefficients

$$L_s = L_{sm} + L_{s\sigma} \quad ; \quad L_r = L_{rm} + L_{r\sigma}$$

we directly find from figure 6 for the induction machine:

$$\vec{\Psi}_{s\alpha\beta} = L_s \vec{i}_{s\alpha\beta} + M C_{rot}(p\theta)\,\vec{i}_{rdq} \quad ; \quad \vec{\Psi}_{rdq} = L_r \vec{i}_{rdq} + C_{rot}(-p\theta) M \vec{i}_s$$

$$\vec{u}_{s\alpha\beta} = R_s \vec{i}_{s\alpha\beta} + \frac{\mathrm{d}\vec{\Psi}_{s\alpha\beta}}{\mathrm{d}t} \quad ; \quad \vec{u}_{rdq} = R_r \vec{i}_{rdq} + \frac{\mathrm{d}\vec{\Psi}_{rdq}}{\mathrm{d}t}$$

We can eliminate the rotation matrices in these equations by expressing the rotor quantities in the stator coordinate system by means of (16):

$$\vec{\Psi}_{s\alpha\beta} = L_s \vec{i}_{s\alpha\beta} + M\vec{i}_{r\alpha\beta} \quad ; \quad \vec{\Psi}_{r\alpha\beta} = M\vec{i}_{s\alpha\beta} + L_r \vec{i}_{r\alpha\beta}$$

$$\vec{u}_{s\alpha\beta} = R_s \vec{i}_{s\alpha\beta} + \frac{\mathrm{d}\vec{\Psi}_{s\alpha\beta}}{\mathrm{d}t} \quad ; \quad \vec{u}_{r\alpha\beta} = R_r \vec{i}_{r\alpha\beta} - p\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\Psi}_{r\alpha\beta} + \frac{\mathrm{d}\vec{\Psi}_{r\alpha\beta}}{\mathrm{d}t}$$

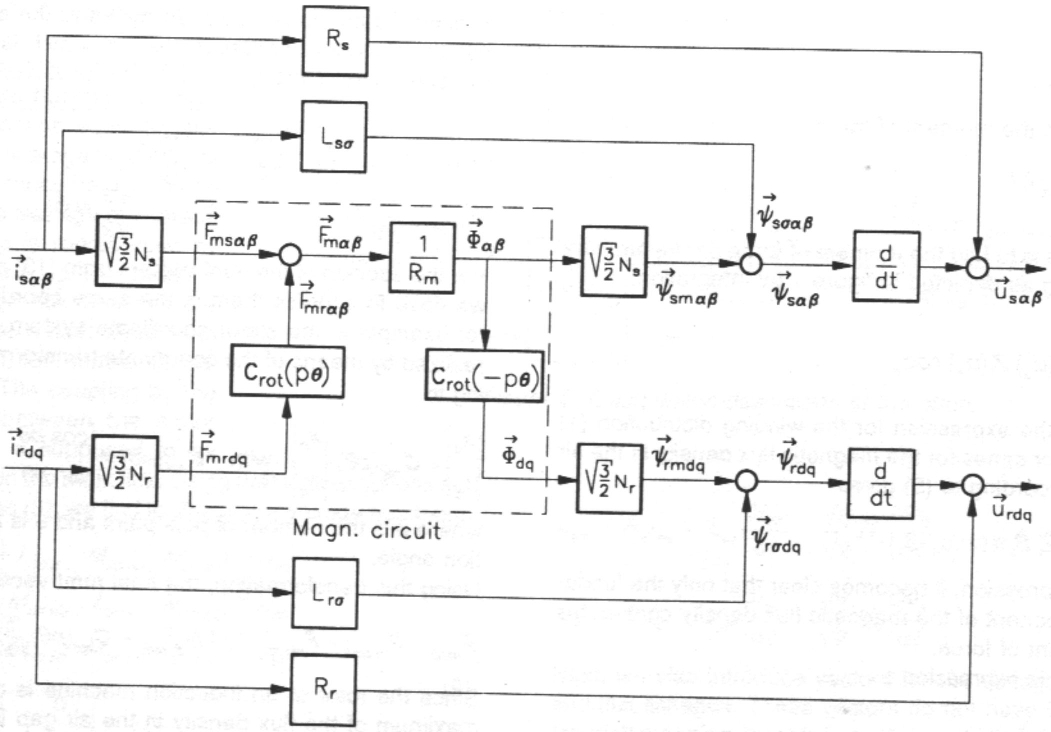

Figure 6 A schematic description of the induction machine

A set of state space equations

This set of equations may directly be written as the set of state space equations

$$\frac{d\vec{\psi}_{s\alpha\beta}}{dt} = \vec{u}_{s\alpha\beta} - R_s\vec{i}_{s\alpha\beta}$$

$$\frac{d\vec{\psi}_{r\alpha\beta}}{dt} = \vec{u}_{r\alpha\beta} - R_r\vec{i}_{r\alpha\beta} + p\omega_m\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\vec{\psi}_{r\alpha\beta} \qquad (19)$$

$$\vec{i}_{s\alpha\beta} = \frac{\vec{\psi}_{s\alpha\beta} - \frac{M}{L_r}\vec{\psi}_{r\alpha\beta}}{\sigma L_s} \; ; \; \vec{i}_{r\alpha\beta} = \frac{\vec{\psi}_{r\alpha\beta} - \frac{M}{L_s}\vec{\psi}_{s\alpha\beta}}{\sigma L_r}$$

where the leakage factor σ is given by

$$\sigma = 1 - \frac{M^2}{L_sL_r}$$

This set of state space equations may be used in any simulation program after splitting the vectors into their components. In the program MATLAB, however, these equations may be used directly, because vectors are automatically incorporated in MATLAB.

When the MATLAB simulation tool box SIMULINK is used, the block diagram in figure 7 is very appropriate. This block diagram directly follows from the equations (19).

In this block diagram an expression for the electromagnetic torque has been incorporated. This expression comes from (14).

Here, the machine equations have only been given for the stator coordinate system. They may easily be transformed to any other coordinate system, like for example the rotor flux coordinate system for field oriented control.

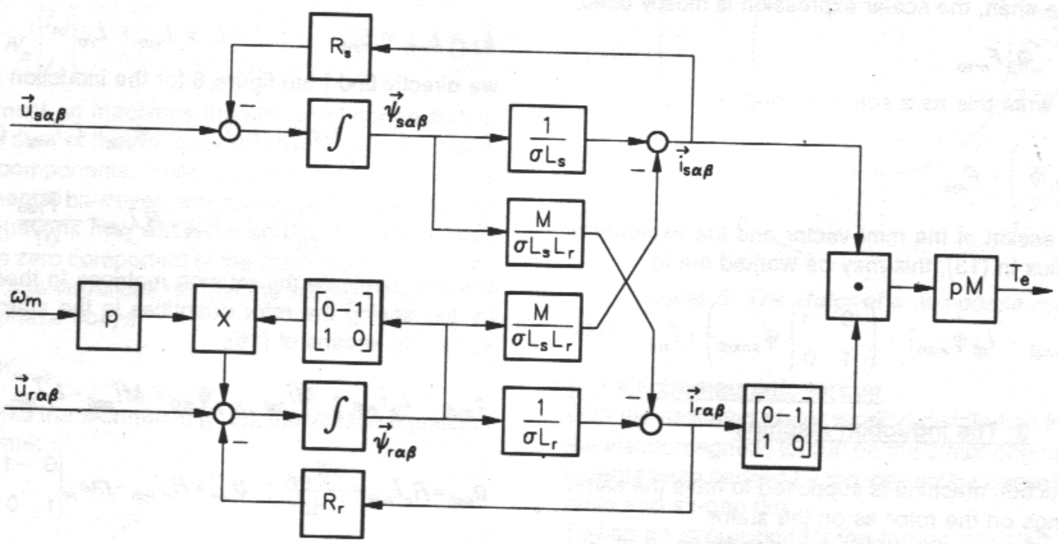

Figure 7 A block diagram of an induction machine

4 The salient-pole synchronous machine

The basic idea of modelling a salient-pole synchronous machine is explained by means of a reluctance machine.
For the stator, the equations (13) may still be used. Since a reluctance machine does not have windings on the rotor, we only have to find the relation between the mmf vector $\vec{F}_m$ and the flux vector $\vec{\Phi}$ in order to get a complete description of the electrical behaviour of a reluctance machine.
When the rotor of a machine is not cylindrical, the maximum of the flux density in the air gap is not at the same place as the maximum of the mmf. Hence, the mmf vector $\vec{F}_m$ and the flux vector $\vec{\Phi}$ do not have the same direction. Here, the mmf vector is resolved into two components (figure 8):

$$\vec{F}_m = \vec{F}_{m,d} + \vec{F}_{m,q}$$

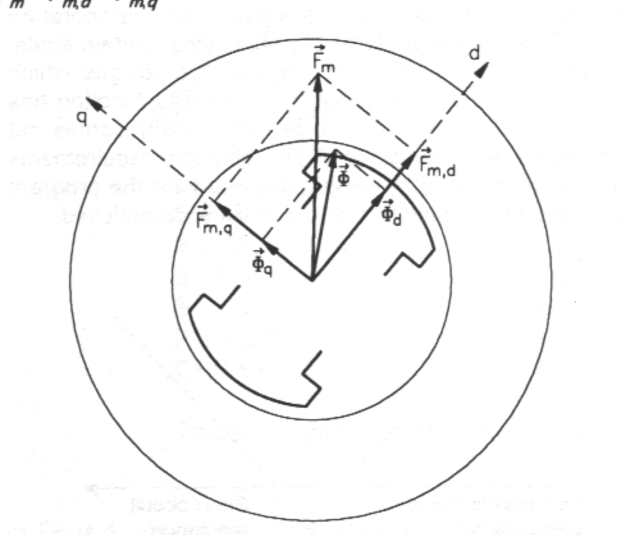

Figure 8 A salient-pole machine

For each of these components the magnetic circuit is symmetric again. Hence, we may use a reluctance for the direct axis and a reluctance for the quadrature axis:

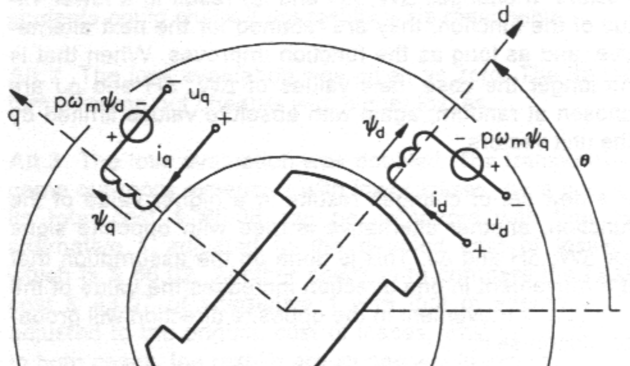

Figure 9 A schematic description of a reluctance machine in the rotor coordinate system

$$\vec{\Phi}_d = \frac{\vec{F}_{m,d}}{R_{md}} \quad ; \quad \vec{\Phi}_q = \frac{\vec{F}_{m,q}}{R_{mq}}$$

As a result, the air-gap flux density may be expressed as

$$\vec{\Phi} = \vec{\Phi}_d + \vec{\Phi}_q = \frac{\vec{F}_{m,d}}{R_{md}} + \frac{\vec{F}_{m,q}}{R_{mq}}$$

Main flux saturation may be incorporated in various ways in the reluctances R_{md} and R_{mq}.
Since we are only interested in the d and q components of the mmf and the flux vector, the rotor coordinate system is the most appropriate system for the description of a salient-pole machine. For that reason we transform the stator equations (13) to the rotor coordinate system by means of (16):

$$\vec{u}_{sdq} = R_s \vec{i}_{sdq} + p\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_{sdq} + \frac{d\vec{\psi}_{sdq}}{dt}$$

$$\vec{\psi}_{sdq} = L_{s\sigma} \vec{i}_{sdq} + \vec{\psi}_{smdq} \qquad (20)$$

$$\vec{F}_{ms,\alpha\beta} = \frac{\sqrt{3}}{\sqrt{2}} N_s \vec{i}_{s\alpha\beta} \quad ; \quad \vec{\psi}_{sm\alpha\beta} = \frac{\sqrt{3}}{\sqrt{2}} N_s \vec{\Phi}_{\alpha\beta}$$

It should be noted that the Park transformation is automatically found by using, successively, the Clarke transformation according to (10) and the rotation transformation according to (16).
After introducing the main flux inductances

$$L_{dm} = \frac{\frac{3}{2} N_s^2}{R_{m,d}} \quad ; \quad L_{qm} = \frac{\frac{3}{2} N_s^2}{R_{m,q}}$$

equation (20) may be combined to a more usual, scalar form:

$$u_d = R_s i_d - p\omega_m \psi_q + \frac{d\psi_d}{dt} \quad ; \quad u_q = R_s i_q + p\omega_m \psi_d + \frac{d\psi_q}{dt}$$

$$\psi_d = L_{s\sigma} i_d + \psi_{dm} = L_{s\sigma} i_d + L_{dm} i_d$$

$$\psi_q = L_{s\sigma} i_q + \psi_{qm} = L_{s\sigma} i_q + L_{qm} i_q$$

The voltage equations are schematically depicted in figure 9.

Conclusion

In this paper, two vectors with a physical meaning for the description of AC machines have been introduced. The first one is a vector representing the magnetomotive force, while the second (new) one represents the fundamental wave of the air-gap flux density.

It has been shown that using these vectors, models of AC machines may easily be derived. Especially for the case of salient poles or main flux saturation, the derivation is much more straight forward than conventional derivations.

References

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