A SYSTEM'S APPROACH OF THE EDDY CURRENTS IN THE ROTOR OF AN INDUCTON MACHINE

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Abstract

The normal way of modelling skin effect is by adding one or more extra rotor windings to the equivalent circuit of the induction machine. Here, operational inductances (transfer functions) are used for modelling skin effect.
In the derivation of the machine equations, the stator windings are modelled in the usual way. On the other hand, the electric circuits (windings and eddy currents) on the rotor are not explicitly modelled, but the rotor is seen as a linear system (with a tranfer function). This system is excited by the magnetomotive force caused by the stator currents, given in the rotor reference frame. Its response is a contribution to the air gap flux, which is transformed to the stator reference frame. Next, it is used to find equations for the stator voltages.
The rotor transfer function may be obtained from a standstill test by means of modern system identification techniques.

Introduction

The frequency of the electrical quantities in the rotor of an induction machine ranges from very low (the normal slip frequency) to normal (the grid frequency in case of conventional starting) or very high (switching frequency of the convertor). This means that the skin effect in the rotor windings should almost always be taken into account in the model of an induction machine.
The normal way of modelling skin effect is by adding an extra rotor winding (without skin effect) to the usual equivalent circuit of the induction machine. When the model does not come up to the expectations, more windings may be added.
The way of modelling by means of adding windings to the equivalent circuit is not quite systematic and it is difficult to find out how many windings are needed and to obtain the winding parameters.
In this paper operational inductances, similar to operational reactances in synchronous machines, are introduced. Because these operational inductances are actually transfer functions in the frequency (s) domain, it is possible to use modern system identification techniques. These system identification techniques include the estimation of the order of the system and the estimation of the parameters.
We begin by deriving the machine equations. In this derivation, the stator windings are modelled in the usual way, using generally accepted assumptions.
On the other hand, the electric circuits (windings and eddy currents) on the rotor are not explicitly modelled, but the rotor is seen as a linear system which is excited by the magnetomotive force caused by the stator currents. This is possible by first transforming the magnetomotive force vector from the stator coordinate system to the rotor coordinate system. The response of the linear system, which represents the currents or current densities on the rotor, is a contribution to the air gap flux given as a vector in the rotor coordinate system. After transforming this flux vector to the stator coordinate system, it is used to find expressions for the flux linkages of the stator windings and the stator voltages.

The problem to identify the transfer function representing the currents or current densities on the rotor may be solved by means of standstill tests, because in this case, the rotation transformation is not active. Hence, it is also possible to describe the whole machine by means of a transfer function, which may directly be identified by modern system identification techniques. After identifying this transfer function, the transfer function representing the currents or current densities on the rotor is derived.

Although the resulting model of the induction machine is complete, it still is difficult to use because it consists of a non-linear part describing the rotation transformation (rotational voltages) in the time domain and a linear part describing the rotor in the frequency (s) domain. The next step is to transform the transfer function for the rotor to a set of state equations by standard techniques and to eliminate the rotation transformation by transforming quantities in the rotor coordinate system to the stator coordinate system. In this way a model of the induction machine is obtained which only exists of (non-linear) state equations, so that it may easily be used in simulation programs like MATLAB/SIMULINK.

1 The basic model

In this section we develop the basic model of the induction machine. The first step is the computation of the magnetomotive force caused by the stator currents. After rotating this magnetomotive force from the stator to the rotor coordinate system, the rotor reaction on this magnetomotive force is modelled by means of a transfer function. This results in an expression for the air gap flux in rotor coordinates, which is transformed to stator coordinates. Finally, the stator voltages follow.

The stator currents as excitation

An important supposition in the modelling is that the stator winding consists of three sinusoidally distributed windings along the stator circumference in the air gap.

We start with observing one stator winding (subscript 1), which is sinusoidally distributed along the stator circumference according to

$$Z_1(\alpha_s) = \hat{Z}_1 \sin(\alpha_s - \alpha_1) \qquad (1)$$

where Z_1 is the number of conductors per metre and α_1 represents the axis of the winding (see figure 1). The angle α_s is the air gap coordinate angle.

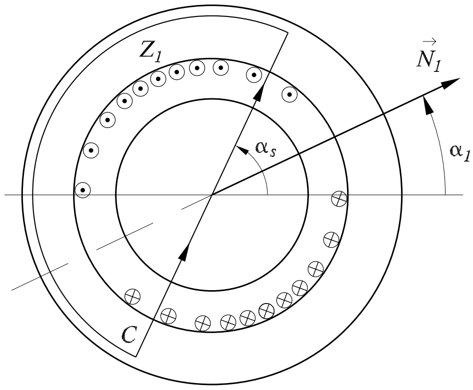

Figure 1 The observed sinusoidally distributed winding

Using Ampere's integral law, an expression for the magnetomotive force caused by the current in the winding i_1 may be found:

$$F_{m,1}(\alpha_s) = i_1 \int_{\alpha_s}^{\alpha_s+\pi} Z_1(\alpha') r \, \mathrm{d}\alpha' = 2 r i_1 \hat{Z}_1 \cos(\alpha_s - \alpha_1)$$

where r is the stator bore radius. This expression may be simplified by using the total number of turns of the winding, which is found by integrating (1):

$$N_1 = r \int_{\alpha_1}^{\alpha_1+\pi} Z_1(\alpha') \mathrm{d}\alpha' = 2 r \hat{Z}_1 \qquad (2)$$

This results in

$$F_{m,1}(\alpha_s) = i_1 N_1 \cos(\alpha_s - \alpha_1)$$

The observed winding may be represented by the vector $\vec{N}_1$, which is in the direction of the axis of the winding and the length of which corresponds with the total number of turns, as is illustrated in figure 1.

Next, we introduce a vector for the magnetomotive force:

$$\vec{F}_{m,1} = i_1 \vec{N}_1 \qquad (3)$$

The length of this vector equals the maximum value of the magnetomotive force (mmf) and the direction of the vector is the direction of the maximum of the magnetomotive force: the mmf is distributed cosinusoidally around this vector.

Now, the mmf caused by a three-phase stator winding may be found by a vector addition (see figure 2):

$$\vec{F}_{ms} = i_{sa}\vec{N}_{sa} + i_{sb}\vec{N}_{sb} + i_{sc}\vec{N}_{sc}$$

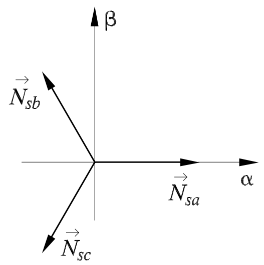

Figure 2 The stator winding vectors

Up to now, the vectors were not represented in a particular coordinate system. When the stator coordinate system is used (with α-axis and β-axis), the vector for the stator mmf is

$$\begin{aligned} \vec{F}_{ms}^{\alpha\beta} = \begin{bmatrix} F_{ms\alpha} \\ F_{ms\beta} \end{bmatrix} &= i_{sa} N_s \begin{bmatrix} 1 \\ 0 \end{bmatrix} + i_{sb} N_s \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2}\sqrt{3} \end{bmatrix} + i_{sc} N_s \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2}\sqrt{3} \end{bmatrix} \\ &= N_s \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2}\sqrt{3} \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} \end{aligned} \qquad (4)$$

The superscript αβ denotes that the αβ coordinate system is used.

The rotor model

The rotor is supposed to be fully rotational symmetric, so that the real rotor position does not matter. This condition is mostly fulfilled because the number of bars in the rotor cage is normally high.

To model the rotor, it is observed from the rotor itself in a coordinate system which is fixed to the rotor. This rotor coordinate system is denoted by the direct (d) and the quadrature (q) axis, as is usual in synchronous machines (figure 3; the dot on the circle is a mark on the rotor to define its position).

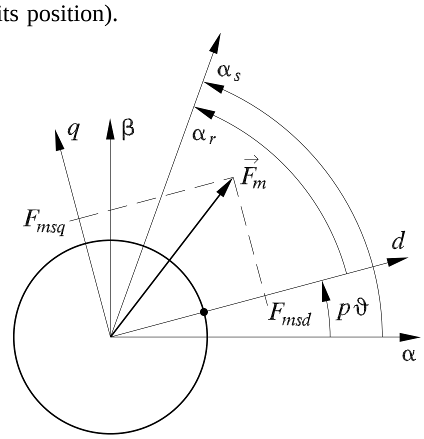

Figure 3 The stator and the rotor coordinate system

Because the stator mmf vector is seen as the excitation for the rotor, it should also be expressed in the rotor coordinate system. This may be realized by means of the coordinate transformation according to

$$\begin{bmatrix} F_{ms\alpha} \\ F_{ms\beta} \end{bmatrix} = \boldsymbol{C}_{rot}(p\theta) \begin{bmatrix} F_{msd} \\ F_{msq} \end{bmatrix} \tag{5}$$

with $\boldsymbol{C}_{rot}(p\theta) = \begin{bmatrix} \cos p\theta & -\sin p\theta \\ \sin p\theta & \cos p\theta \end{bmatrix}$

where p is the number of pole pairs and θ is the rotor position angle (see figure 3).

First, we consider the direct-axis component of the mmf vector in the rotor coordinate system. In general, this component is a function of time: $F_{msd}(t)$. This changing vector component results in a current distribution in the rotor (which opposes its cause). As a result of this current distribution and the stator magnetomotive force $F_{msd}(t)$, there is a flux density in the air gap: $B_d(t,\alpha_r)$, where α_r is the rotor coordinate (see figure 3).

In general, the spatial distribution of $B_d(t,\alpha_r)$ is not sinusoidal. However, because the stator windings are sinusoidally distributed, only the fundamental component of the spatial distribution gives a contribution to the flux linkages of the stator windings (a more formal proof may be found in [1]). Therefore, only this component is considered further. Thanks to symmetry properties, it may be expressed as

$$B_{1d}(t,\alpha_r) = \hat{B}_{1d}(t)\cos\alpha_r$$

Since eddy current phenomena in the rotor have a linear behaviour, these phenomena may be represented by a linear system with $F_{msd}(t)$ as input and $\hat{B}_{1d}(t)$ as output. Hence, the Laplace transform may be used and the electromagnetic behaviour of the rotor may be described by means of a transfer function ($H(s)$). The Laplace transform of the amplitude of the flux density distribution is given by:

$$\hat{B}_{1d}(s) = H(s)\, F_{msd}(s) \tag{6}$$

For the quadrature axis a similar expression is found. Next, the expressions for the direct and for the quadrature axis may be combined to a vector expression:

$$\begin{bmatrix} \hat{B}_{1d}(s) \\ \hat{B}_{1q}(s) \end{bmatrix} = H(s) \begin{bmatrix} F_{msd}(s) \\ F_{msq}(s) \end{bmatrix} \quad \text{or} \quad \vec{\hat{B}}_1^{dq}(s) = H(s)\, \vec{F}_{ms}^{dq}(s) \tag{7}$$

This vector points in the direction of the maximum of the fundamental wave of the flux density in the air gap.

Some remarks on the transfer function

We may write the transfer function in the following form:

$$H(s) = H_0 \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \tag{8}$$

In the case of a very fast changing mmf vector (the Laplace variable is very high: $s=\infty$), the fundamental wave of the flux density is relatively small, but not zero (there is always some leakage flux in the air gap). Hence, the number of poles and the number of zeros in the transfer function have to be equal.

In the following we use a second order transfer functions as an example. So, (7) may be written as

$$\vec{\hat{B}}_1^{dq}(s) = H_0 \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....}\, \vec{F}_{ms}^{dq}(s) \tag{9}$$

The main flux linkages of the stator windings

To find the flux linkages of the stator windings, we start to consider the contribution of the direct-axis component $\hat{B}_{1d}(t)$ to the flux linkage of phase winding a [1]:

$$\psi_{sma,d} = \frac{\pi}{2} l r N_s \hat{B}_{1,d} \cos p\theta \tag{10}$$

where l is the core length. Using the vector notation for the stator windings, this equation may be written as a scalar product of the flux density vector and the winding vector:

$$\psi_{sma,d} = \frac{\pi}{2} l r \vec{\hat{B}}_{1,d} \cdot \vec{N}_{sa} \tag{11}$$

Here, we used that $p\theta$ is the angle between the vector of the fundamental wave of the air gap flux density (only d-component) and the winding axis.

When the quadrature component is considered too, (11) may be generalized to

$$\psi_{sma} = \frac{\pi}{2} l r \vec{\hat{B}}_1 \cdot \vec{N}_{sa} \tag{12}$$

We may see the area $\frac{\pi}{2} lr$ as a kind of area of the magnetic circuit for the air gap flux. So, we may introduce a vector for the air gap flux:

$$\vec{\Phi} = \frac{\pi}{2} l r \vec{\hat{B}}_1 \tag{13}$$

Now, (12) may be written as

$$\psi_{sma} = \vec{\Phi} \cdot \vec{N}_{sa} \tag{14}$$

Hence, the fluxes linked with the three stator windings may now be given by (see figure 2):

$$\psi_{sma} = \vec{N}_{sa} \cdot \vec{\Phi} = N_s \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \Phi_\alpha \\ \Phi_\beta \end{bmatrix} = N_s \Phi_\alpha$$

$$\psi_{smb} = \vec{N}_{sb} \cdot \vec{\Phi} = N_s \left(-\frac{1}{2}\Phi_\alpha + \frac{\sqrt{3}}{2}\Phi_\beta \right)$$

$$\psi_{smc} = \vec{N}_{sc} \cdot \vec{\Phi} = N_s \left(-\frac{1}{2}\Phi_\alpha - \frac{\sqrt{3}}{2}\Phi_\beta \right)$$

or

$$\begin{bmatrix} \psi_{sma} \\ \psi_{smb} \\ \psi_{smc} \end{bmatrix} = N_s \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \end{bmatrix} \begin{bmatrix} \Phi_\alpha \\ \Phi_\beta \end{bmatrix} \tag{15}$$

For the development of these expressions the stator (αβ) coordinate system has been used.

The stator winding leakage flux

To find the stator winding voltages, the leakage flux has also to be taken into account. The leakage flux of one phase winding is represented by the inductance $L_{s\sigma a}$. The coupling by the leakage flux between the phase windings is accounted for in the coefficient for mutual inductance $M_{s\sigma ab}$. The coupling by the leakage flux between the stator and the rotor is supposed to be accounted for in the main flux. Using these assumptions, we find for the stator flux linkages:

$$\begin{bmatrix}\psi_{sa}\\ \psi_{sb}\\ \psi_{sc}\end{bmatrix} = \begin{bmatrix}L_{s\sigma a} & M_{s\sigma ab} & M_{s\sigma ab}\\ M_{s\sigma ab} & L_{s\sigma a} & M_{s\sigma ab}\\ M_{s\sigma ab} & M_{s\sigma ab} & L_{s\sigma a}\end{bmatrix}\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{sc}\end{bmatrix} + \begin{bmatrix}\psi_{sma}\\ \psi_{smb}\\ \psi_{smc}\end{bmatrix} \qquad (16)$$

The stator winding resistances and leakage flux

These fluxes may be used for the stator voltage equations:

$$\begin{bmatrix}u_{sa}\\ u_{sb}\\ u_{sc}\end{bmatrix} = R_s\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{sc}\end{bmatrix} + \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\psi_{sa}\\ \psi_{sb}\\ \psi_{sc}\end{bmatrix} \qquad (17)$$

The complete model

In fact, we now have a complete model of the induction machine. This model consists of the equations (4), (5), (9), (13), (15), (16), and (17).

2 The Clarke transformation

The model equations may be simplified by means of the Clarke transformation. This simplification is based on the fact that the air-gap behaviour of the machine is determined by the two components of the vectors $\vec{F}_m$ and $\vec{\Phi}$, as may be seen in the equations (4) and (15).

Here, we use the normalized Clarke transformation:

$$\begin{bmatrix}x_{s\alpha}\\ x_{s\beta}\\ x_{s0}\end{bmatrix} = \boldsymbol{C}_{\alpha\beta 0,abc}\begin{bmatrix}x_{sa}\\ x_{sb}\\ x_{sc}\end{bmatrix} \qquad (18)$$

$$\text{with} \quad \boldsymbol{C}_{\alpha\beta 0,abc} = \frac{\sqrt{2}}{\sqrt{3}}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2}\end{bmatrix}$$

The basic elements of the transformation matrix automatically arise from the equations (4) and (15).

Using the Clarke transformation, the expression for the mmf vector (4) is simplified to:

$$\vec{F}_{ms}^{\alpha\beta} = \sqrt{\tfrac{3}{2}}N_s\begin{bmatrix}i_{s\alpha}\\ i_{s\beta}\end{bmatrix} \qquad (19)$$

and the expression for the main flux linkages of the stator windings (15) is simplified to

$$\begin{bmatrix}\psi_{sm\alpha}\\ \psi_{sm\beta}\end{bmatrix} = \sqrt{\tfrac{3}{2}}N_s\vec{\Phi}^{\alpha\beta} \qquad (20)$$

Because in most ac machines the star point connection is not used, there are no zero components. Here, we pay no attention to the zero components. However, when they are of interest, the belonging equations may easily be added.

Using that the zero component of the stator currents is zero, we see in equations (16) that the leakage inductance seen in one phase obeys

$$L_{s\sigma} = L_{s\sigma a} - M_{s\sigma ab}$$

Using the Clarke transformation ((18)), the expression for the stator flux linkages (16) now becomes:

$$\begin{bmatrix}\psi_{s\alpha}\\ \psi_{s\beta}\end{bmatrix} = L_{s\sigma}\begin{bmatrix}i_{s\alpha}\\ i_{s\beta}\end{bmatrix} + \begin{bmatrix}\psi_{sm\alpha}\\ \psi_{sm\beta}\end{bmatrix} \qquad (21)$$

Space vectors

The vectors with the α and the β components of the voltages, currents, and flux linkages are called space vectors. These vectors do not have any physical (spatial) meaning, what is in contrast with the vectors $\vec{\Phi}$ and $\vec{F}_m$. To get a more compact description of the induction machine these vectors are written like

$$\vec{x}^{\alpha\beta} = \begin{bmatrix}x_\alpha\\ x_\beta\end{bmatrix} \qquad (22)$$

If such a vector is rotated by means of the coordinate transformation according to (5), it gets the form

$$\vec{x}^{dq} = \begin{bmatrix}x_d\\ x_q\end{bmatrix} \qquad (23)$$

The equations (19), (20), and (21) may now be written in a compact form:

$$\vec{F}_{ms}^{\alpha\beta} = \sqrt{\tfrac{3}{2}}N_s\,\vec{i}_s^{\alpha\beta} \qquad (24)$$

$$\vec{\psi}_{sm}^{\alpha\beta} = \sqrt{\tfrac{3}{2}}N_s\vec{\Phi}^{\alpha\beta} \qquad (25)$$

$$\vec{\psi}_s^{\alpha\beta} = L_{s\sigma}\vec{i}_s^{\alpha\beta} + \vec{\psi}_{sm}^{\alpha\beta} \qquad (26)$$

Using the Clarke transformation ((18)), the voltage equations (17) now become (there is no zero component):

$$\vec{u}_s^{\alpha\beta} = R_s\vec{i}_s^{\alpha\beta} + \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta}}{\mathrm{d}t} \qquad (27)$$

The complete model

Except for the Clarke transformation ((18)), the complete model of the induction machine now consists of the following steps:

1. computation of the stator magnetomotive force ((24));
2. rotation of the stator magnetomotive force from the $\alpha\beta$ to the *dq* coordinate system ((5));
3. computation of the flux density by means of the transfer function ((9));
4. computation of the air gap flux vector ((13));
5. rotation of the air gap flux vector from the *dq* to the $\alpha\beta$ coordinate system ((5));
6. computation of the stator main flux linkages ((25));
7. computation of the stator flux linkages ((26));
8. computation of the stator voltage equations ((27)).

3 The operational inductances

The next step in simplifying the set of equations, is combining the equations (9) and (13) in rotor coordinates (the steps 3 and 4)

$$\vec{\Phi}^{dq}(s) = \frac{\pi}{2} lr \vec{B}_1^{dq}(s) = \frac{\pi}{2} lr H_0 \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \vec{F}_{ms}^{dq}(s) \quad (28)$$

In correspondence with Hopkinson's Law, we may now introduce the magnetic reluctance R_m for the case s=0, according to

$$\frac{1}{R_m} = \frac{\pi}{2} lr H_0$$

Now, we may write (28) as

$$\vec{\Phi}^{dq}(s) = \frac{1}{R_m} \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \vec{F}_{ms}^{dq}(s) \quad (29)$$

After transforming the expression for the stator magnetomotive force (24) and the expression for the main flux linkages (25) to rotor coordinates by using (5), these equations may be combined with (29) to

$$\begin{aligned} \vec{\Psi}_{sm}^{dq}(s) &= \sqrt{\tfrac{3}{2}} N_s \vec{\Phi}^{dq}(s) \\ &= \sqrt{\tfrac{3}{2}} N_s \frac{1}{R_m} \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \vec{F}_{ms}^{dq}(s) \\ &= \sqrt{\tfrac{3}{2}} N_s \frac{1}{R_m} \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \sqrt{\tfrac{3}{2}} N_s \vec{I}_s^{dq}(s) \end{aligned} \quad (30)$$

Here, capitals are used to indicate Laplace transformed quantities.

After introducing the main inductance L_{sm} according to

$$L_{sm} = \frac{\frac{3}{2} N_s^2}{R_m}$$

the equation for the main flux (30) becomes

$$\vec{\Psi}_{sm}^{dq}(s) = L_{sm} \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \vec{I}_s^{dq}(s) \quad (31)$$

After transforming the expression for the flux linkages (26) to rotor coordinates by using (5), this equation may be combined with (31) to

$$\begin{aligned} \vec{\Psi}_s^{dq}(s) &= L_{s\sigma} \vec{I}_s^{dq}(s) + \vec{\Psi}_{sm}^{dq}(s) \\ &= \left(L_{s\sigma} + L_{sm} \frac{(1+s\tau_1^{/})(1+s\tau_1^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \right) \vec{I}_s^{dq}(s) \end{aligned}$$

This expression is written in the form

$$\vec{\Psi}_s^{dq}(s) = L_s(s) \vec{I}_s^{dq}(s) = L_s \frac{(1+s\tau^{/})(1+s\tau^{//}).....}{(1+s\tau_0^{/})(1+s\tau_0^{//}).....} \vec{I}_s^{dq}(s) \quad (32)$$

In this equation, the operational inductance $L_s(s)$ has been introduced.

This operational inductance is similar to the well-known operational inductances (reactances) of synchronous machines [2].

The complete model

Except for the rotation and the Clarke transformation ((5) and (18)), the complete model of the induction machine now consists of the stator voltage equation (27) and the flux expression (32). It should be noted that the voltage equation (27) is given in stator coordinates and in the time domain, where the flux equation (32) is given in rotor coordinates and in the frequncy (s) domain.

4 System identification by means of standstill tests

For system identification standstill tests may easily be used. Since the rotor is supposed to be cilindrical symmetric, we may choose θ=0. This means that the rotor and the stator coordinate system are the same and that we may directly substitute the flux equation (32) into the Laplace transformed stator voltage equation (27):

$$\vec{U}_s^{\alpha\beta}(s) = (R_s + sL_s(s)) \vec{I}_s^{\alpha\beta}(s)$$

After measuring the stator resistance R_s and using the Clarke transformation ((5)), the operational inductance $L_s(s)$ may directly be determined by means of modern system identification techniques.

5 The simulation model

From frequency domain to time domain

Since it is the intention to make a model for digital simulation, the operational inductance, which is a transfer function, is transformed into state equations.

To find the corresponding state equations (time domain), we first expand the transfer function (32) into partial fractions:

$$\vec{\Psi}_s^{dq} = L_\sigma \vec{I}_s^{dq} + \tau_0^{/} \frac{R_s^{/} \vec{I}_s^{dq}}{1+s\tau_0^{/}} + \tau_0^{//} \frac{R_s^{//} \vec{I}_s^{dq}}{1+s\tau_0^{//}} + ... \quad (33)$$

Here the coefficients L_σ, $R_s^{/}$, and $R_s^{//}$ have been introduced, which follow directly from expanding the transfer function. The coefficient L_σ is given by

$$L_\sigma = \frac{\tau^{/}\tau^{//}.....}{\tau_0^{/}\tau_0^{//}.....} L_s$$

This inductance is seen when the stator current (in rotor coordinates) is changing very rapidly (the Laplace variable s is very large). In the case of a second order system, this may be called the subtransient inductance

$$L_s^{//} = \frac{\tau^{/}\tau^{//}}{\tau_0^{/}\tau_0^{//}} L_s$$

Now, we introduce the fluxes:

$$\vec{\Psi}_s^{dq\,/} = \tau_0^{/} \frac{R_s^{/} I_s^{dq}}{1+s\tau_0^{/}} \quad ; \quad \vec{\Psi}_s^{dq\,//} = \tau_0^{//} \frac{R_s^{//} I_s^{dq}}{1+s\tau_0^{//}} \quad (34)$$

Using these expressions, the Laplace transform of the stator flux vector Ψ_s according to (33) may be written as

$$\vec{\Psi}_s^{dq} = L_\sigma \vec{I}_s^{dq} + \vec{\Psi}_s^{dq\,/} + \vec{\Psi}_s^{dq\,//} + ... \qquad (35)$$

Next, the equations (34) and (35) are transformed to the time domain:

$$\begin{aligned} \frac{\mathrm{d}\vec{\psi}_s^{dq\,/}}{\mathrm{d}t} &= -\frac{\vec{\psi}_s^{dq\,/}}{\tau_0^/} + R_s^/ \vec{i}_s^{dq} \\ \frac{\mathrm{d}\vec{\psi}_s^{dq\,//}}{\mathrm{d}t} &= -\frac{\vec{\psi}_s^{dq\,//}}{\tau_0^{//}} + R_s^{//} \vec{i}_s^{dq} \end{aligned} \qquad (36)$$

$$\vec{\psi}_s^{dq} = L_\sigma \vec{i}_s^{dq} + \vec{\psi}_s^{dq\,/} + \vec{\psi}_s^{dq\,//} + ... \qquad (37)$$

As we may see, equation (34) passes into the set of state equations (36), which may directly be used in a simulation program.

The transformation from the rotor to the stator coordinate system

The next step is the transformation of the equations (36) and (37) to the stator coordinate system. This is realized by multiplying these equations by $\boldsymbol{C}_{rot}(p\theta)$ according to (5) and transforming the vectors:

$$\begin{aligned} \boldsymbol{C}_{rot}(p\theta)\frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{C}_{rot}(p\theta)\,\vec{\psi}_s^{\alpha\beta/}\right) &= -\frac{\vec{\psi}_s^{\alpha\beta/}}{\tau_0^/} + R_s^/ \vec{i}_s^{\alpha\beta} \\ \boldsymbol{C}_{rot}(p\theta)\frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{C}_{rot}(p\theta)\,\vec{\psi}_s^{\alpha\beta//}\right) &= -\frac{\vec{\psi}_s^{\alpha\beta//}}{\tau_0^{//}} + R_s^{//} \vec{i}_s^{\alpha\beta} \end{aligned} \qquad (38)$$

$$\vec{\psi}_s^{\alpha\beta} = L_\sigma \vec{i}_s^{\alpha\beta} + \vec{\psi}_s^{\alpha\beta/} + \vec{\psi}_s^{\alpha\beta//} + ... \qquad (39)$$

Further, using (5), we develop the left term of the equations (38):

$$\begin{aligned} \boldsymbol{C}_{rot}(p\theta)\frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{C}_{rot}(-p\theta)\vec{x}^{\alpha\beta}\right) &= -\boldsymbol{C}_{rot}(p\theta)p\frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\mathrm{d}\boldsymbol{C}_{rot}(-p\theta)}{\mathrm{d}(-p\theta)}\vec{x}^{\alpha\beta} \\ &+ \boldsymbol{C}_{rot}(p\theta)\boldsymbol{C}_{rot}(-p\theta)\frac{\mathrm{d}\vec{x}^{\alpha\beta}}{\mathrm{d}t} \end{aligned}$$

After introducing the mechanical angular speed

$$\omega_m = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad (40)$$

this equations becomes

$$\boldsymbol{C}_{rot}(p\theta)\frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{C}_{rot}(-p\theta)\vec{x}^{\alpha\beta}\right) = -p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{x}^{\alpha\beta} + \frac{\mathrm{d}\vec{x}^{\alpha\beta}}{\mathrm{d}t}$$

When we use this result in (38), we find

$$\begin{aligned} -p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_s^{\alpha\beta/} + \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta/}}{\mathrm{d}t} &= -\frac{\vec{\psi}_s^{\alpha\beta/}}{\tau_0^/} + R_s^/ \vec{i}_s^{\alpha\beta} \\ -p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_s^{\alpha\beta//} + \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta//}}{\mathrm{d}t} &= -\frac{\vec{\psi}_s^{\alpha\beta//}}{\tau_0^{//}} + R_s^{//} \vec{i}_s^{\alpha\beta} \end{aligned} \qquad (41)$$

The square matrix in these equations may be seen as a rotation over an angle $\pi/2$. The corresponding (new) term in the equations may be seen as a rotational voltage.

The stator voltage equation

The next step is to substitute the expression for the stator flux (39) into the stator voltage equation (27):

$$\vec{u}_s^{\alpha\beta} = R_s \vec{i}_s^{\alpha\beta} + L_\sigma \frac{\mathrm{d}\vec{i}_s^{\alpha\beta}}{\mathrm{d}t} + \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta/}}{\mathrm{d}t} + \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta//}}{\mathrm{d}t} + ...$$

After substituting the flux derivatives from (41) into this equation, this stator voltage equation may be written as

$$\begin{aligned} \vec{u}_s^{\alpha\beta} = R_s \vec{i}_s^{\alpha\beta} + L_\sigma \frac{\mathrm{d}\vec{i}_s^{\alpha\beta}}{\mathrm{d}t} - \frac{\vec{\psi}_s^{\alpha\beta/}}{\tau_0^/} + p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_s^{\alpha\beta/} + \\ - \frac{\vec{\psi}_s^{\alpha\beta//}}{\tau_0^{//}} + p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_s^{\alpha\beta//} + R_s^{//} \vec{i}_s^{\alpha\beta} + ... \end{aligned} \qquad (42)$$

The complete model

Except for Clarke transformation ((5)), the complete model of the induction machine now consists of the stator voltage equation (42) and the flux (differential) equations (41). When we write the equations (42) and (41) in a state space form, we find

$$\begin{aligned} L_\sigma \frac{\mathrm{d}\vec{i}_s^{\alpha\beta}}{\mathrm{d}t} = \vec{u}_s^{\alpha\beta} - \left(R_s + R_s^/ + R_s^{//} + ...\right)\vec{i}_s^{\alpha\beta} + \frac{\vec{\psi}_s^{\alpha\beta/}}{\tau_0^/} + \\ - p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \left(\vec{\psi}_s^{\alpha\beta/} + \vec{\psi}_s^{\alpha\beta//} + ...\right) \end{aligned} \qquad (43)$$

$$\begin{aligned} \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta/}}{\mathrm{d}t} &= -\frac{\vec{\psi}_s^{\alpha\beta/}}{\tau_0^/} + p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_s^{\alpha\beta/} + R_s^/ \vec{i}_s^{\alpha\beta} \\ \frac{\mathrm{d}\vec{\psi}_s^{\alpha\beta//}}{\mathrm{d}t} &= -\frac{\vec{\psi}_s^{\alpha\beta//}}{\tau_0^{//}} + p\,\omega_m \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_s^{\alpha\beta//} + R_s^{//} \vec{i}_s^{\alpha\beta} \end{aligned} \qquad (44)$$

Because a simulation program like MATLAB/SIMULINK can use vector variables directly, these equations may directly be used.

Conclusion

A model of an induction machine has been derived which is completely characterized by the stator resistance R_s and a transfer function, the operational inductance $L_s(s)$. This operational inductance may be obtained from a standstill test by means of modern system identification techniques. Besides, it has been shown that this model may be expressed as a (compact) set of state equations by expanding the operational inductance into partial fractions and transforming the quantities given in the rotor frame of reference to the stator frame of reference.

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