# HARMONIC ANALYSIS OF A PM MACHINE WITH RECTIFIER

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**Abstract**

This paper describes a method to calculate the steady-state performance of a permanent-magnet (PM) generator with six-pulse controlled rectifier without considering the transient interval. The calculation method solves the machine equations by means of Fourier analysis.

This calculation method uses frequency-dependent operational inductances (operational impedances), which are determined by means of simple locked-rotor tests. Using these operational inductances has the advantage, that the total losses (including iron losses and eddy current losses in magnets) due to the harmonics of the stator currents are calculated rather accurately. These losses form a considerable part of the total losses, and they are seriously underestimated if only the copper losses are considered.

Measurements show the validity of the calculation method.

**Keywords**

permanent-magnet machine, controlled rectifier, harmonic analysis, operational inductance, hybrid vehicle

## 1 Introduction

The aim of the research project[1] is the development of a high speed, high efficiency generator system, intended for use in series-hybrid vehicles, the drive system of which is depicted in figure 1. The combustion engine is a high-speed gas turbine, the advantage of which is that it has lower emissions than other combustion engines. The generator is a permanent-magnet (PM) machine, because of its high efficiency, reliability, high power density, and the possibilities for high speed. For the same reasons, the rectifier is a six-pulse controlled rectifier.

The generator system can also be applied in aircraft, vessels, and mobile ground power stations.

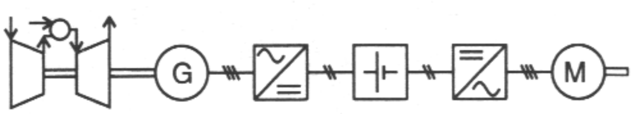


*Figure 1: The drive system of a series-hybrid car, consisting of gas turbine, PM generator, rectifier, accumulator, inverter and motor.*

This paper describes a method to calculate the steady-state performance of the PM generator with rectifier, depicted in figure 2. The calculation method, introduced in [1]-[3], solves the machine equations by means of Fourier analysis.

Two important advantageous characteristics of this method are the following. First, the steady-state performance can be calculated without considering the transient interval. Second, frequency-dependent machine parameters can be used.

This paper adds the following four significant aspects to this calculation method.

1) As in [4], it is proposed to use measured, frequency-dependent operational inductances, which has two important advantages:
   - It is not necessary to derive frequency-dependent machine parameters from (complicated) machine models, as is done in [2]-[3]. Instead, we use easily measured operational inductances.
   - The losses due to the harmonics of the stator currents are calculated rather accurately, including the losses due to eddy currents in iron and magnets.
2) For this method, it is necessary to split the machine voltages into a voltage across an external inductance and an internal voltage. In [1]-[3], no attention is paid to the choice of this external inductance. This paper introduces a sensible choice of this external inductance.
3) The calculation method uses the no-load voltage. It is proposed to use the measured no-load voltage (which is not sinusoidal), instead of a calculated sinusoidal no-load voltage as in [2]-[4].
4) This paper shows that this calculation method can also be used for PM machines.

The method enables calculations with small inductances in the DC-circuit [1]-[4]. However, in this paper we consider the case with an infinitely large inductance in the DC-circuit, because this is enough to describe the principles of the calculation method.

As might be expected of a calculation method for synchronous machines, the method considers different direct-axis and quadrature-axis inductances [2]-[4]. However, the PM machine has a cylindrical rotor with surface-mounted magnets. Therefore, the difference between the inductances of the direct-axis and the quadrature-axis is small and is neglected in this paper.

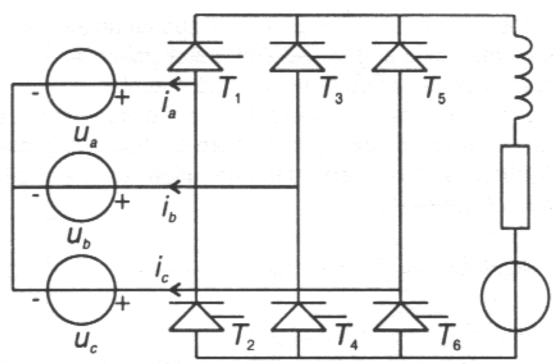


*Figure 2: The PM generator with controlled rectifier.*

[1]This research project is sponsored by NOVEM (Netherlands Agency for energy and the Environment)

### Structure of the paper

This paper has the following structure. First, section 2 introduces the voltage equations of the PM machine using operational inductances. Next, section 3 describes the determination of the operational inductances of the PM machine by means of measurements. Further, section 4 describes the calculation method. In section 5, the results of the calculation method are compared to a measurement. We close in section 6 with some conclusions.

## 2 Voltage equations with operational inductances

The expression for voltage $u$ of the stator phases $a$, $b$, and $c$ is given by

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} e_{pa} \\ e_{pb} \\ e_{pc} \end{bmatrix} + R_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} \tag{1}$$

where

$e_p$ is the no-load voltage,

$i$ is the phase current,

$R_s$ is the resistance of a stator phase winding, and

$\psi$ is the part of the flux linked with a stator phase winding dependent on the stator currents.

In this equation, the flux linked with the stator phase windings was split into two parts:

1) A part $\psi$ dependent on the stator currents. This part includes the flux due to the stator currents, but also the fluxes due to the rotor currents which are induced by the stator currents. These rotor currents are eddy currents and damper currents if a damper would be present.
2) A part independent from the stator currents and dependent on the magnetization of the magnets. The time derivative of this part is the no-load voltage $e_p$.

In the rest of this paper, the measured no-load voltage $e_p$ (which is not sinusoidal) is used for the no-load voltage. The stator quantities can be rotated to the rotor connected $dq$-system by means of the well-known Park transformation [5]. In its normalized form, this transformation is given by

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} ; \begin{bmatrix} e_{pd} \\ e_{pq} \\ e_{p0} \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} e_{pa} \\ e_{pb} \\ e_{pc} \end{bmatrix} ; \begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} ; \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} ;$$

$$\text{with} \quad \boldsymbol{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(p\theta) & \cos(p\theta - \frac{2}{3}\pi) & \cos(p\theta - \frac{4}{3}\pi) \\ -\sin(p\theta) & -\sin(p\theta - \frac{2}{3}\pi) & -\sin(p\theta - \frac{4}{3}\pi) \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix} \tag{2}$$

where

$\theta$ is the rotor position angle or the spatial angle between the direct-axis and the axis of stator phase $a$, and

$p$ is the number of pole-pairs of the machine.

Because there is no star-point connection, the zero-components are always zero and are omitted in this paper. Application of the Park transformation to the voltage equation (1), results in

$$\begin{aligned} u_d &= e_{pd} + R_s i_d + \frac{d\psi_d}{dt} - p\omega_m \psi_q \\ u_q &= e_{pq} + R_s i_q + \frac{d\psi_q}{dt} + p\omega_m \psi_d \end{aligned} \tag{3}$$

where $\omega_m$ is the constant angular speed of the rotor.

Using the Fourier transformation, this equation changes into

$$\begin{aligned} \underline{u}_d(\omega) &= \underline{e}_{pd}(\omega) + R_s \underline{i}_d(\omega) + j\omega\underline{\psi}_d(\omega) - p\omega_m \underline{\psi}_q(\omega) \\ \underline{u}_q(\omega) &= \underline{e}_{pq}(\omega) + R_s \underline{i}_q(\omega) + j\omega\underline{\psi}_q(\omega) + p\omega_m \underline{\psi}_d(\omega) \end{aligned} \tag{4}$$

The fluxes in this equation can be written as

$$\begin{aligned} \underline{\psi}_d(\omega) &= \underline{L}_d(\omega)\, \underline{i}_d(\omega) \\ \underline{\psi}_q(\omega) &= \underline{L}_q(\omega)\, \underline{i}_q(\omega) \end{aligned} \tag{5}$$

where

$\underline{L}_d(\omega)$ is the direct-axis operational inductance, and

$\underline{L}_q(\omega)$ is the quadrature-axis operational inductance.

Substitution of this expression in equation (4), gives

$$\begin{aligned} \underline{u}_d(\omega) &= \underline{e}_{pd}(\omega) + \{R_s + j\omega\underline{L}_d(\omega)\}\underline{i}_d(\omega) - p\omega_m \underline{L}_q(\omega)\underline{i}_q(\omega) \\ \underline{u}_q(\omega) &= \underline{e}_{pq}(\omega) + \{R_s + j\omega\underline{L}_q(\omega)\}\underline{i}_q(\omega) + p\omega_m \underline{L}_d(\omega)\underline{i}_d(\omega) \end{aligned} \tag{6}$$

The imaginary part of the operational inductances represents the rotor losses. It is assumed that the stator is without losses, except for the stator copper losses.

As described in [4] and [5], operational impedances are used since 1929. In this paper and in [4], the term operational inductance is preferred and used, because these operational inductances give a relation between the current and the flux.

## 3 Determination of operational inductances

### Locked-rotor voltage equations

The operational inductances are determined by means of locked-rotor tests. If the machine does not rotate, $\omega_m=0$ is valid and the no-load voltage $e_p$ is zero. If this is used in the voltage equation (6), the voltage equation becomes:

$$\begin{aligned} \underline{u}_d(\omega) &= \{R_s + j\omega\underline{L}_d(\omega)\}\underline{i}_d(\omega) \\ \underline{u}_q(\omega) &= \{R_s + j\omega\underline{L}_q(\omega)\}\underline{i}_q(\omega) \end{aligned} \tag{7}$$

### Direct-axis operational inductance

For the determination of the direct-axis operational inductance $\underline{L}_d(\omega)$, the rotor is placed so that the quadrature-axis coincides with the axis of stator phase $a$ ($p\theta = \pi/2$). A voltage is supplied to the stator phases $b$ and $c$, which are connected in series as depicted in figure 3. In this way, the direct-axis coincides with the armature winding field axis.

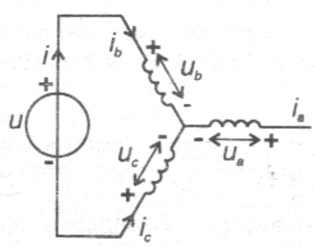


*Figure 3: The measurement circuit for the determination of the operational inductances.*

During the measurements, $i_b = i$, $i_c = -i$, and $u_b - u_c = u$ (figure 3). With the Park transformation (2), the $dq$-components of the currents and voltages are calculated. If these $dq$-components are substituted in equation (7), we obtain

$$\begin{aligned} \underline{u}(\omega) &= 2\{R_s + j\omega\underline{L}_d(\omega)\}\underline{i}(\omega) \\ &= 2\{R_s - \omega\,\mathrm{Im}(\underline{L}_d(\omega)) + j\omega\,\mathrm{Re}(\underline{L}_d(\omega))\}\underline{i}(\omega) \\ &= \{R_d(\omega) + j\omega L_d(\omega)\}\underline{i}(\omega) \end{aligned} \tag{8}$$

where

$$R_d(\omega) = 2R_s - 2\omega \mathrm{Im}(L_d(\omega)) \quad (9)$$

$$L_d(\omega) = 2\mathrm{Re}(L_d(\omega)) \quad (10)$$

So, the determination of the direct-axis operational inductance $L_d(\omega)$ comes to the determination of an impedance $R_d(\omega)+j\omega L_d(\omega)$. To determine this impedance, the measuring circuit of figure 3 is supplied by a sinusoidal voltage source, the frequency and the amplitude of which are adjustable. The applied voltage $U$, the current $I$, and the active power $P$ are measured. From this, the impedance can be calculated.

**Quadrature-axis operational inductance**

For the determination of the quadrature-axis operational inductance $L_q(\omega)$, the rotor is placed so that the direct-axis coincides with the axis of phase $a$ ($p\theta=0$). A voltage is supplied to the stator phases $b$ and $c$, which are connected in series as depicted in figure 3. In this way, the quadrature-axis coincides with the armature winding field axis.

In the same way as for the determination of the direct-axis operational inductance, the determination of the quadrature-axis operational inductance comes to the determination of an impedance $R_q(\omega)+j\omega L_q(\omega)$. This impedance is measured in the same way as for the direct-axis operational inductance.

**Discussion of the results**

Figure 4 gives the measured impedances in the direct- and quadrature-axis as a function of the frequency for different current amplitudes.

As can be seen in figure 4, the direct-axis operational inductance is a little smaller than the quadrature-axis operational inductance. However, the differences between the direct-axis and the quadrature-axis operational inductances are small, because the machine has a cylindrical rotor with surface-mounted magnets. Therefore, as already mentioned in the Introduction (section 1), in the rest of this paper, it is assumed that the direct-axis and the quadrature-axis operational inductances are equal:

$$L_d(\omega) = L_q(\omega) = L(\omega) \quad (11)$$

The value of the operational inductance $L(\omega)$ is determined by taking the average of the measured values for the direct-axis and the quadrature-axis operational inductance.

At each frequency, the voltage, the current, and the power were measured for different root-mean-square values of the current. This was done, because hysteresis may cause different results at different values of the current. As can be seen in figure 4, there are differences: the larger the amplitude of the current, the larger the measured inductance. The amplitudes of the currents during rectifier operation are large. Therefore, in the calculation of the steady-state performance of the machine with rectifier, the values measured at the largest current amplitude are used. The lines in figure 4 represent the values of the impedance used in the calculation method.

As mentioned in section 2, the imaginary part of the operational inductance represents the rotor losses. In the PM machine, the rotor losses consist of iron losses and eddy current losses in the magnets. However, it should be realized that a part of the measured losses arises in the stator iron and contributes to the imaginary part of the operational inductance.

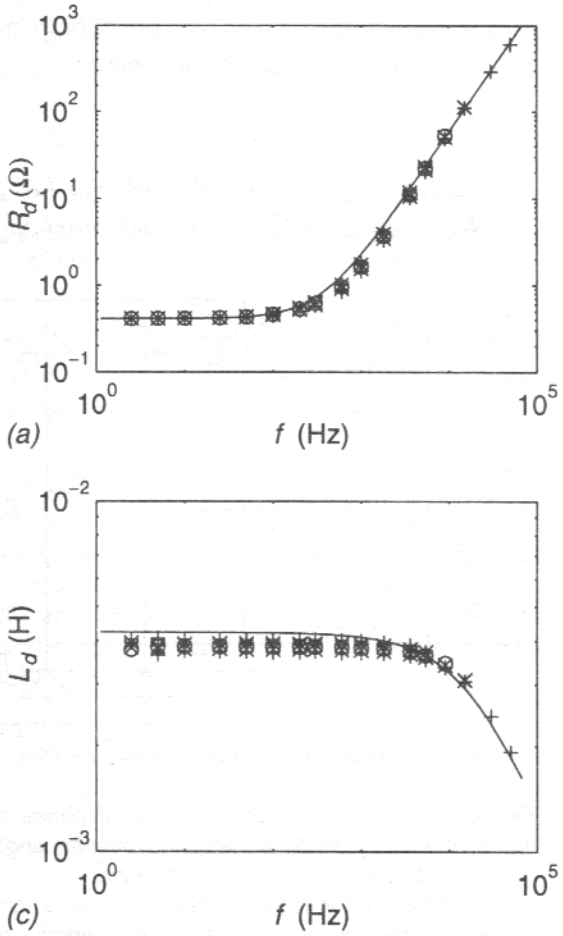


*Figure 4: Measured impedance as a function of the frequency in the direct- and quadrature-axis for different currents (+: I=0.2A, x: I=0.5A, o: I=1A, *: I=2A, +: I=5A), and the values used in the calculation method (——).*

## 4 Calculation method

Before explaining the calculation method, this section describes the used Fourier series and steady-state voltage equations. It closes with a discussion about the choice of the external inductance.

### Fourier series

In the calculation method described in this paper, Fourier series are used in the following way.
If $f(t)$ is a periodic function of time with period $T$, it can be written as a Fourier series with Fourier coefficients $\hat{f}_n$:

$$f(t) = \sum_{n=-\infty}^{\infty} \hat{f}_n e^{jn\omega_1 t} \quad \text{with} \quad \hat{f}_n = \frac{1}{T}\int_{t=x}^{x+T} f(t) e^{-jn\omega_1 t} dt \tag{12}$$

where $\omega_1 = 2\pi/T$ is the angular frequency of the fundamental component of the periodic signal $f(t)$.

### Steady-state voltage equations using Fourier series

The calculation method considers the steady-state performance of a PM machine with rectifier. This implies the following three things.

1) The rotor rotates synchronously:

$$p\omega_m = \omega_1 \tag{13}$$

2) The stator three stator currents are equal except for a time shift:

$$i_a(\omega_1 t) = i_b(\omega_1 t - \tfrac{2}{3}\pi) = i_c(\omega_1 t - \tfrac{4}{3}\pi) \tag{14}$$

This is also valid for the voltages and the fluxes.

3) The currents, the fluxes and the voltages do not contain even harmonics, because there is half wave symmetry. They also do not contain harmonics of which the harmonic number is an integer multiple of three, because there is no star-point connection. Therefore, the current and the voltage of phase $a$ can be written as the following Fourier series:

$$i_a = \sum_{n=-\infty}^{\infty} \hat{\underline{i}}_{a,6n+1} e^{j(6n+1)\omega_1 t} + \hat{\underline{i}}_{a,6n-1} e^{j(6n-1)\omega_1 t} \tag{15}$$

Using (14), the currents of the phases $b$ and $c$ are expressed as Fourier series with the Fourier coefficients of the current of phase $a$. These currents are rotated to the $dq$-system by means of the Park transformation (2). The result is a Fourier series for the current in the $dq$-system, which only contains harmonics of which the harmonic number is an integer multiple of six:

$$i_d = \sum_{n=-\infty}^{\infty} \hat{\underline{i}}_{d,6n} e^{j6n\omega_1 t} \; ; \quad i_q = \sum_{n=-\infty}^{\infty} \hat{\underline{i}}_{q,6n} e^{j6n\omega_1 t} \tag{16}$$

where

$$\hat{\underline{i}}_{d,6n} = \sqrt{\tfrac{3}{2}}(\hat{\underline{i}}_{a,6n+1} + \hat{\underline{i}}_{a,6n-1}) \; ; \quad \hat{\underline{i}}_{q,6n} = \sqrt{\tfrac{3}{2}}(\hat{\underline{i}}_{a,6n+1} - \hat{\underline{i}}_{a,6n-1}) \tag{17}$$

This is also valid for the voltages and the fluxes.

In the voltage equation in the $dq$-system (3), all quantities are written as Fourier series, while it is used that this equation only contains harmonics of which the harmonic number is an integer multiple of six:

$$u_d = \sum_{n=-\infty}^{\infty} \hat{\underline{u}}_{d,6n} e^{j6n\omega_1 t} \quad ; \quad u_q = \sum_{n=-\infty}^{\infty} \hat{\underline{u}}_{q,6n} e^{j6n\omega_1 t} \tag{18}$$

where

$$\begin{aligned} \hat{\underline{u}}_{d,6n} &= \hat{\underline{e}}_{pd,6n} + R_s \hat{\underline{i}}_{d,6n} + j6n\omega_1 \hat{\underline{\Psi}}_{d,6n} - \omega_1 \hat{\underline{\Psi}}_{q,6n} \\ \hat{\underline{u}}_{q,6n} &= \hat{\underline{e}}_{pq,6n} + R_s \hat{\underline{i}}_{q,6n} + j6n\omega_1 \hat{\underline{\Psi}}_{q,6n} + \omega_1 \hat{\underline{\Psi}}_{d,6n} \end{aligned} \tag{19}$$

The fluxes in this equation can be replaced by operational inductances multiplied by currents (as in (6)):

$$\begin{aligned} \hat{\underline{u}}_{d,6n} &= \hat{\underline{e}}_{pd,6n} + \{R_s + j6n\omega_1 \underline{L}(6n\omega_1)\}\hat{\underline{i}}_{d,6n} - \omega_1 \underline{L}(6n\omega_1)\hat{\underline{i}}_{q,6n} \\ \hat{\underline{u}}_{q,6n} &= \hat{\underline{e}}_{pq,6n} + \{R_s + j6n\omega_1 \underline{L}(6n\omega_1)\}\hat{\underline{i}}_{q,6n} + \omega_1 \underline{L}(6n\omega_1)\hat{\underline{i}}_{d,6n} \end{aligned} \tag{20}$$

If these Fourier coefficients are substituted in the Fourier series of equation (18), and equation (18) is transformed back with the inverse Park transformation (2), the resulting voltage of phase $a$ is:

$$\begin{aligned} u_a &= \sum_{n=-\infty}^{\infty} \hat{\underline{u}}_{a,6n+1} e^{j(6n+1)\omega_1 t} + \hat{\underline{u}}_{a,6n-1} e^{j(6n-1)\omega_1 t} \; ; \\ \text{with } \hat{\underline{u}}_{a,6n+1} &= \hat{\underline{e}}_{pa,6n+1} + \{R_s + j(6n+1)\omega\, \underline{L}(6n\omega_1)\}\hat{\underline{i}}_{a,6n+1} \\ \hat{\underline{u}}_{a,6n-1} &= \hat{\underline{e}}_{pa,6n-1} + \{R_s + j(6n-1)\omega\, \underline{L}(6n\omega_1)\}\hat{\underline{i}}_{a,6n-1} \end{aligned} \tag{21}$$

### Description of the calculation method

Figure 5, which depicts a schematic drawing of the PM machine with controlled rectifier, is used for the explanation of the calculation method.
Figure 5 depicts a current source $I_{DC}$ in the DC-circuit, because we consider the case with an infinitely large inductance in the DC-circuit (as mentioned in section 1).
For the calculation method, it is necessary to split the machine voltage into a voltage over an external inductance $L_e$ and an internal voltage $e$, as indicated in figure 5:

$$e_a = u_a - L_e \frac{di_a}{dt} \tag{22}$$

If the Fourier series for the current (15) and the voltage (21) are substituted in this equation, we obtain

$$\begin{aligned} e_a &= \sum_{n=-\infty}^{\infty} \hat{\underline{e}}_{a,6n+1} e^{j(6n+1)\omega_1 t} + \hat{\underline{e}}_{a,6n-1} e^{j(6n-1)\omega_1 t} \; ; \\ \text{with } \hat{\underline{e}}_{a,6n+1} &= \hat{\underline{e}}_{pa,6n+1} + \{R_s + j(6n+1)\omega(\underline{L}(6n\omega_1) - L_e)\}\hat{\underline{i}}_{a,6n+1} \\ \hat{\underline{e}}_{a,6n-1} &= \hat{\underline{e}}_{pa,6n-1} + \{R_s + j(6n-1)\omega(\underline{L}(6n\omega_1) - L_e)\}\hat{\underline{i}}_{a,6n-1} \end{aligned} \tag{23}$$

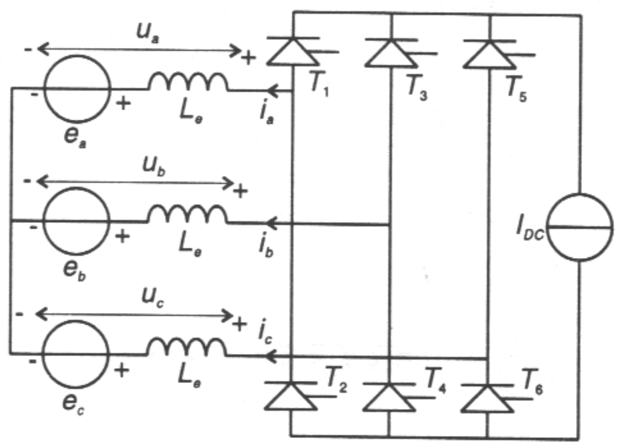


*Figure 5: Model of the PM machine with rectifier.*

Figure 6 depicts a stylized current $i_a$ of phase $a$. In this figure, the delay angle $\alpha_p$ and the overlap angle $\mu$ are given, which are constant in steady-state.
Steady-state performance implies that it is enough to consider one period. During this period, phase $a$ commutates four times. In figure 6, the four commutation

intervals are numbered I to IV. Table 1 gives the time derivative of the current $i_a$ during the commutation intervals. Table 1 also shows which thyristors are conducting during the four commutation intervals. If phase $a$ is not commutating, the time derivative of the current $i_a$ is zero, because of the infinitely large inductance in the DC-circuit.

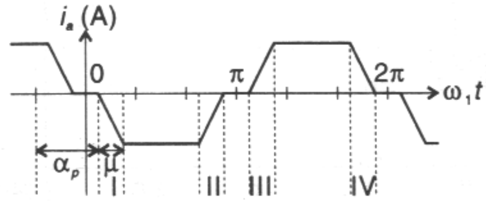


*Figure 6: The stylized current $i_a$ of phase a and the commutation intervals.*

*Table 1: The time derivative of the current during the commutation intervals.*

| Interval | | Conducting thyristors | $2L_e\frac{di_a}{dt}$ |
|---|---|---|---|
| I | $\alpha_p-\frac{1}{3}\pi\leq\omega_1 t<\alpha_p+\mu-\frac{1}{3}\pi$ | $T_1$, $T_5$, $T_4$ | $e_a-e_c$ |
| II | $\alpha_p+\frac{1}{3}\pi\leq\omega_1 t<\alpha_p+\mu+\frac{1}{3}\pi$ | $T_1$, $T_3$, $T_6$ | $e_a-e_b$ |
| III | $\alpha_p+\frac{2}{3}\pi\leq\omega_1 t<\alpha_p+\mu+\frac{2}{3}\pi$ | $T_3$, $T_2$, $T_6$ | $e_a-e_c$ |
| IV | $\alpha_p+\frac{4}{3}\pi\leq\omega_1 t<\alpha_p+\mu+\frac{4}{3}\pi$ | $T_5$, $T_2$, $T_4$ | $e_a-e_b$ |

Using table 1, the time derivative of $i_a$ is given by

$$2L_e\frac{di_a}{dt} = (e_a-e_c)\left(p(\omega_1 t)+p(\omega_1 t-\pi)\right) + (e_a-e_b)\left(p(\omega_1 t-\tfrac{2}{3}\pi)+p(\omega_1 t-\tfrac{5}{3}\pi)\right) \qquad (24)$$

where the pulse function $p(\omega_1 t)$ is 1 during commutation interval I of table 1 and zero everywhere else:

$$p(\omega_1 t) = \begin{cases} 1 & \text{when } -\frac{1}{3}\pi+\alpha_p<\omega_1 t<-\frac{1}{3}\pi+\alpha_p+\mu \\ 0 & \text{when } -\frac{1}{3}\pi+\alpha_p+\mu<\omega_1 t<\frac{5}{3}\pi+\alpha_p \end{cases} \qquad (25)$$

which can be written as a Fourier series:

$$p = \sum_{n=-\infty}^{\infty}\hat{p}_n e^{jn\omega_1 t} \quad \text{with} \quad \hat{p}_n = \frac{1}{T}\int_{t=0}^{T} p\, e^{-jn\omega_1 t}dt \qquad (26)$$

In equation (24), the current, the internal voltages, and the pulse function are replaced by Fourier series (the equations (15), (23), and (26) respectively). The result is an equation with multiplications of Fourier series (convolutions). From this, the harmonics of the current are determined as

$$\begin{aligned} 2j(6n+1)\omega L_d \hat{i}_{a,6n+1} &= 6\sum_{k=-\infty}^{\infty}\left\{\hat{e}_{a,6k+1}\hat{p}_{6(n-k)}+\hat{e}_{a,6k-1}e^{j\frac{1}{3}\pi}\hat{p}_{6(n-k)+2}\right\} \\ 2j(6n-1)\omega L_d \hat{i}_{a,6n-1} &= 6\sum_{k=-\infty}^{\infty}\left\{\hat{e}_{a,6k-1}\hat{p}_{6(n-k)}+\hat{e}_{a,6k+1}e^{-j\frac{1}{3}\pi}\hat{p}_{6(n-k)-2}\right\} \end{aligned} \qquad (27)$$

which follows after some calculation work.

If the Fourier coefficients of equation (23) are substituted in this equation, the only unknowns in this equation are the Fourier coefficients of the current. For a finite number of harmonics, the set of equations can be solved, because the number of equations is equal to the number of unknown Fourier coefficients of the current.

In this calculation method, it is assumed that the delay angle $\alpha_p$ and the overlap angle $\mu$ are known; if this is not the case, it is necessary to iterate to the right values.

**On the choice of the external inductance $L_e$**

In [1]-[3], no attention is paid to the choice of the external inductance. The external inductance $L_e$ should be chosen equal to the commutation inductance. If this is done, the term $L(6n\omega_1)-L_e$ in equation (23) is very small for the important harmonics. This improves the numerical accuracy of the calculation process.

If this external inductance is used, the internal voltage $e$ becomes a smooth function of time. All jumps in the terminal voltage due to commutation are included in the voltage drop across the external inductance $L_e$. In the extreme case that $R_s$=0 and $L(6n\omega_1)-L_e$=0 are valid for all frequencies, the internal voltage $e$ becomes equal to the no-load voltage $e_p$. In this case, the harmonics of the current can be solved direct from equation (27), which results in a very accurate calculation process.

## 5 Results

**Measured wave-forms**

The measured frequency-dependent operational inductances (depicted in figure 4) were used in the calculation of the steady-state performance of the machine with rectifier. Figure 7 depicts both measurements and calculations of phase current and line voltage waveforms. Both for the line voltage and the phase current the agreement is very good, which shows the validity of the calculation method.

**Losses due to harmonics**

As mentioned in section 2, the imaginary part of the operational inductance represents the rotor losses. This means that the losses during operation with rectifier are calculated on the assumption that all losses (except for the stator copper losses) arise in the rotor. However, during the measurements of the operational inductances, a part of the losses arises in the stator iron and contributes to the imaginary part of the operational inductance. Therefore, using operational inductances to calculate the losses, gives inaccurate results.

The reason that these results are inaccurate is that the frequencies experienced by the stator and the rotor are equal during the measurements, while they are different during operation with rectifier.

However, for the harmonics of the stator currents during rectifier operation, the frequencies experienced by the stator and the rotor are nearly equal. Therefore, the losses due to the harmonics (including iron losses and eddy current losses in the magnets) are calculated with a small error.

For the fundamental component during rectifier operation, the frequency experienced by the rotor is zero, while the frequency experienced by the stator is the fundamental frequency. Therefore, the iron losses in the stator due to the permanent magnet excitation and due to the fundamental component of the stator currents are not calculated. However, literature, e.g. [6], gives methods to calculate these losses.

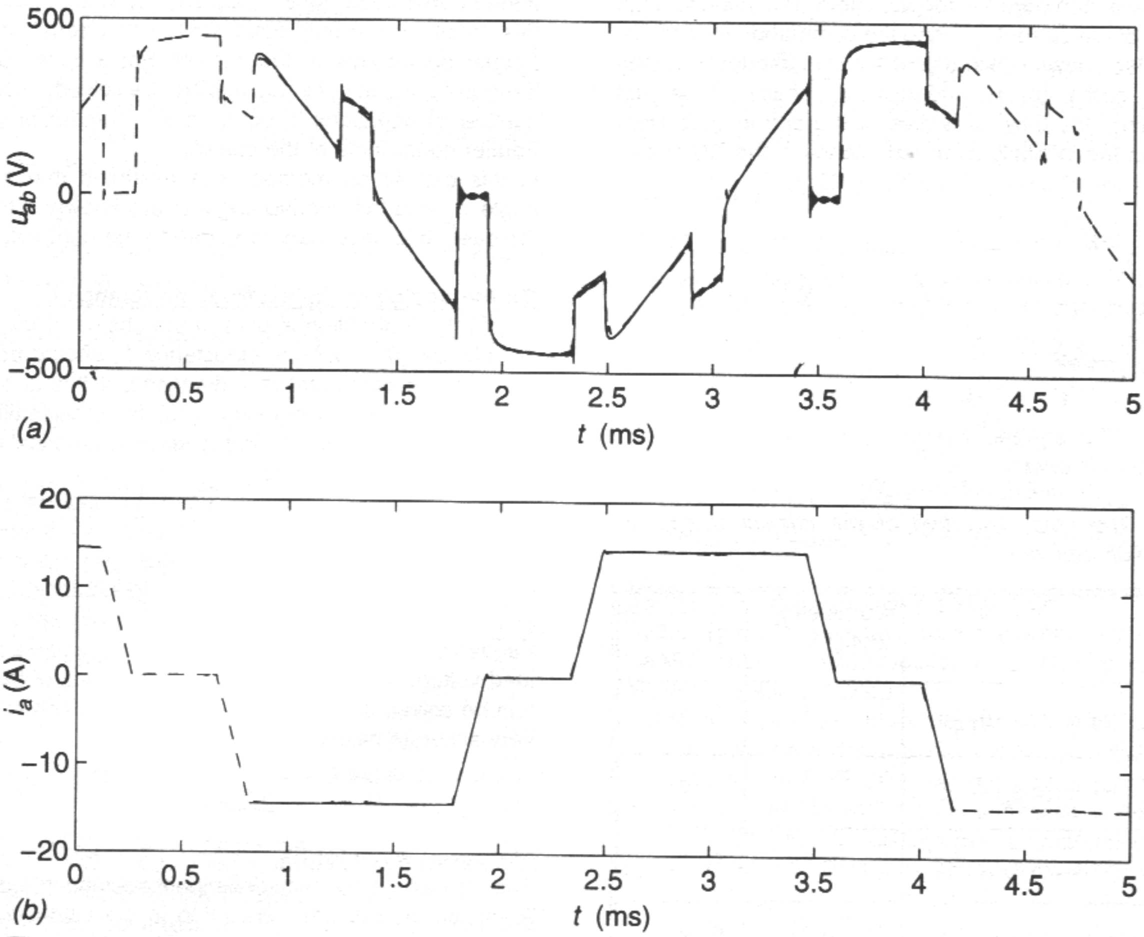


*Figure 7: The measured and one period of the calculated line voltage $u_{ab} = u_a - u_b$ (figure a) and phase current $i_a$ (figure b). The measurements are drawn dashed (- -), the calculations solid (——).*

With the calculation method described in this paper, the losses due to the harmonics of the stator currents in the situation of figure 7 are calculated as 63 W. For comparison: the copper losses due to the harmonics of the stator currents are calculated as 4.2 W. This shows that the losses due to the harmonics of the stator currents in a PM machine are seriously underestimated if only the copper losses are considered. The big difference is formed by the iron losses and the eddy current losses in the magnets, which are considered if operational inductances are used. The copper losses due to the fundamental components of the stator currents are calculated as 82 W. This shows that the losses due to the harmonics of the stator currents form a considerable part of the losses.

## 6 Conclusions

A method to calculate the steady-state performance of a PM machine with controlled rectifier is introduced. This method has the important advantage that it uses frequency-dependent operational inductances, which can be determined by means of simple measurements.
With this method, the losses (including the iron losses and the eddy current losses in the magnets) due to the harmonics of the stator currents are determined rather accurately. During steady-state operation of a PM machine with rectifier, these losses form a considerable part of the losses, and are seriously underestimated if only the copper losses are considered.
For this method, it is necessary to split the machine voltages into a voltage across an external inductance and an internal voltage. It is sensible to choose the external inductance equal to the commutation inductance.
Measurements of current and voltage waveforms show the validity of the calculation method.

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