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Analytic Calculation of the Magnetic Field in PM Machines

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***Abstract* - The aim of the research project this paper arises from, is the development of a high-speed, high-efficiency generator system for use in series-hybrid vehicles. This paper describes an analytic method for the two-dimensional calculation of the magnetic field in the cylindrical air gap of the permanent-magnet generator. Further, it introduces a method to visualize this field, namely by plotting the lines of constant magnetic vector potential. To show that the calculated magnetic field may form the basis of a machine model, the voltage equations are derived. The proposed analytic method gives insight into the relations between dimensions and parameters of the machine, which is important when optimizing the design.**

I. INTRODUCTION

A. The research project

This paper describes a part of a research project, the aim of which is the development of a gas turbine driven high-speed, high-efficiency generator system. This generator system is intended for use in series-hybrid vehicles. Figure 1 depicts the drive system of a series-hybrid vehicle. The generator system can also be applied, for example, in aircraft, in vessels, in mobile ground power stations, and in total energy units.

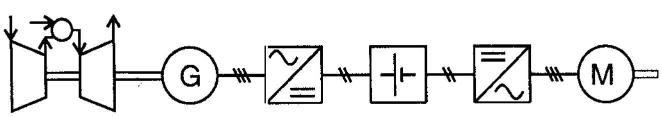

Figure 1: The drive system of a series-hybrid vehicle, consisting of gas turbine, PM generator, rectifier, accumulator, inverter and motor.

The generator system consists of a permanent-magnet (PM) generator with surface-mounted magnets and a six-pulse controlled bridge rectifier, because of their high efficiency, high reliability, and high power density. A fibre bandage surrounding the rotor keeps the magnets in place, which enables high speeds.

B. Objective of the paper

For the optimization of the design and for the analysis of the performance of the generator, a suitable machine model is necessary. This machine model may be based on the calculation of the magnetic field in the air gap of the machine, which is the subject of this paper.
The objective of this paper is to introduce an analytic method for the calculation and visualisation of the magnetic field in the air gap of a PM machine. The derivation of the voltage equations shows that the calculated magnetic field may form the basis of a machine model.
The magnetic field is calculated two-dimensionally and in a cylindrical coordinate system, which means that both the radial and the tangential component are calculated. This is done because the machine is cylindrical, and because the effective air gap (which includes the magnets) is rather large. Further, the space harmonics of the field are considered. In this way, the magnetic field is determined rather accurately. This is done, because later in the research project, the calculated magnetic field will be used to calculate losses in the machine, among which the eddy current loss in the magnets and the loss in a damper cylinder surrounding the rotor.
It is not new to calculate the magnetic field two-dimensionally and in a cylindrical coordinate system, as appears from [1], [2], and [3]. However, these references use the scalar magnetic potential to calculate the magnetic field, and they do not plot lines of magnetic

flux. In this paper, we use the magnetic vector potential. This has the advantage that lines of magnetic flux can easily be plotted as lines of constant magnetic vector potential.
Compared to Finite Element Methods, this method has the disadvantage that it is not suitable for complicated constructions. Therefore, it uses more simplifying assumptions. However, this method has the advantage that it gives more insight in the relations between dimensions and parameters. This is important when the machine design is optimized later in the research project.

C. Main assumptions and starting-points

The derivations in this paper are based on the following assumptions.

- End effects are negligible.
- Hysteresis and eddy currents in stator and rotor iron are negligible, and the magnetic permeability of the iron is infinite.
- Eddy currents the magnets are negligible, and the relative magnetic permeability of the magnets is one.
- Effects of stator slots are negligible. The slotted stator is replaced by a smooth cylindrical stator surface. The current in a stator slot is replaced by a surface (or linear) current density on the stator surface at the place of the slot opening.
- In practice, the magnet pole arcs often consist of rectangular magnet blocks. To make the calculations possible, the magnet blocks are replaced by magnet pole arcs with a radial magnetization which is inversely proportional to the radius.

Figure 2 depicts the PM machine with some important dimensions.

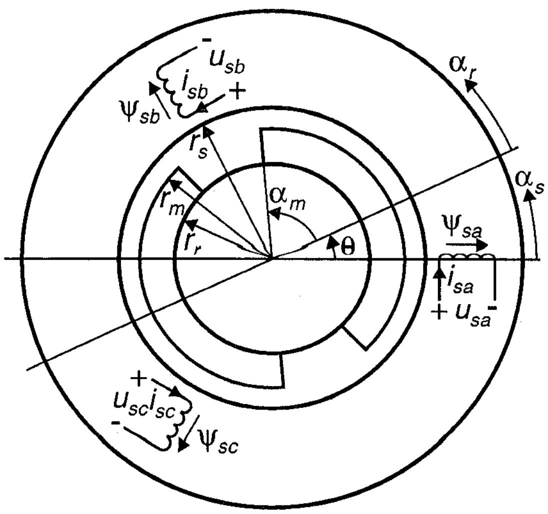

Figure 2: Section of a two-pole PM machine.

D. Outline

This paper has the following structure. First, section II derives a method to calculate and visualize the magnetic field in the air gap of the PM generator. Next, in section III and IV, this method is used to calculate and visualize the magnetic fields due to the magnets and the stator currents. The total magnetic field is a superposition of the contributions due to the magnets and due to the stator currents, because the magnetic circuit is assumed to be linear. Section V shows that the calculated magnetic field may form the basis of a machine model by deriving the voltage equations. Conclusions are drawn in section VI.

II. DERIVATION OF THE CALCULATION METHOD

In this section, the differential equation for the magnetic field in the air gap and the magnets of the machine is derived. To solve this differential equation, boundary conditions are required, which are also given. Further, it is explained how the magnetic field is visualized. For a more elaborate explanation of these derivations is referred to a book, for example, [4].

A. The differential equation

The magnetic field in the air gap and the magnets follows from Maxwell's equations for magnetoquasistatic fields. The equations forming the basis of this derivation are given by

$$\nabla \times \vec{H} = \vec{J} \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \tag{3}$$

where
$\vec{H}$ is the magnetic field strength,
$\vec{J}$ is the current density,
$\vec{B}$ is the magnetic flux density,
μ_0 is the magnetic permeability in vacuum, and
$\vec{M}$ is the magnetization.
Using equation (3) in equation (1) results in

$$\nabla \times \vec{B} = \mu_0(\vec{J} + \nabla \times \vec{M}) \tag{4}$$

These equations become simpler when the magnetic vector potential $\vec{A}$ is used. The relation between the magnetic flux density and the magnetic vector potential is given by

$$\vec{B} = \nabla \times \vec{A} \tag{5}$$

To determine the magnetic vector potential $\vec{A}$

completely, it is not enough to define its rotation. Therefore, its divergence is also defined, and it is chosen as

$$\nabla\cdot\vec{A} = 0 \tag{6}$$

The magnetic vector potential of equation (5) always satisfies equation (2), because the divergence of a rotation is always zero.
Substitution of equation (5) in equation (4) results in a differential equation for the magnetic vector potential $\vec{A}$, which is further worked out using equation (6):

$$\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A} = -\nabla^2\vec{A} = \mu_0(\vec{J}+\nabla\times\vec{M}) \tag{7}$$

In the right side of this differential equation, we use two assumptions:
- the stator currents only have an (axial) z-component which does not depend on z, and
- the magnetization only has a radial component.

With these assumptions, the right side of equation (7) only has a z-component which does not depend on z. Therefore, the magnetic vector potential also only has a z-component which does not depend on z, and the differential equation (equation (7)) can be written as

$$\nabla^2 A_z(r,\alpha) = \begin{cases} \dfrac{\mu_0}{r}\dfrac{\partial M_r}{\partial\alpha} & \text{in the magnets} \\ 0 & \text{in the air spaces} \end{cases} \tag{8}$$

where r and α are the radial coordinate and the angular coordinate of the cylindrical coordinate system.
This differential equation can be solved by means of separation of variables.
When this differential equation is solved, the magnetic field is known, because the magnetic flux density can be calculated by using $\vec{B}=\nabla\times\vec{A}$ (equation (5)).

B. The boundary conditions

On the borders of magnets, air spaces and stator and rotor iron, boundary conditions are required to solve the differential equation.
The magnetic flux continuity condition prescribes that the magnetic flux density normal to a surface between medium a and medium b is continuous:

$$\vec{n}\cdot(\vec{B}^a-\vec{B}^b) = 0 \tag{9}$$

Ampère's continuity condition prescribes that there is a jump in the tangential component of the magnetic field strength as one passes through a surface current density K between medium a and medium b:

$$\vec{n}\times(\vec{H}^a-\vec{H}^b) = K \tag{10}$$

Often, the medium of one of the two regions is iron. Because the magnetic permeability of iron is assumed to be infinite, the magnetic field strength in iron is zero, and the magnetic flux density in iron cannot be determined. In this case, the magnetic flux continuity condition cannot be used, and Ampère's continuity condition is sufficient. These continuity conditions are not boundary conditions for the magnetic vector potential. However, they can be used, because the relations between the magnetic vector potential and the magnetic field strength and the magnetic flux density are known (the equations (5) and (3)).

C. Visualization of the field

A beautiful property of the magnetic vector potential is that it can be used to plot the lines of magnetic flux. This is shown in this subsection, and used in the next sections.
From the magnetic vector potential, the magnetic flux density follows with equation (5). For a two-dimensional field, this equation can be worked out to

$$\vec{B} = \nabla\times\vec{A} = -\vec{i}_z\times\nabla A_z \tag{11}$$

where $\vec{i}_z$ is the unit vector in the z direction.
This shows that the lines of magnetic flux density are perpendicular to the gradient of A_z. Therefore, the lines of magnetic flux can be plotted as the lines of constant magnetic vector potential (or the equipotential lines of the magnetic vector potential).

III. THE MAGNETS

This section starts with a description of the magnetization of the magnets in cylindrical coordinates. This description is used for the calculation of the magnetic field due to the magnets (while the stator currents are zero).

A. The magnetization of the magnets

In practice, the magnet pole arcs often consist of many rectangular magnet blocks with magnetization M_m. To make the calculations possible, the magnet blocks are replaced by magnet pole arcs. The magnetization of these pole arcs only has a radial component M_r, which is inversely proportional to the radius.
Often, the magnet pole arcs do not cover the whole rotor surface, so that there are air spaces between the magnet pole arcs. These air spaces are treated as magnet arcs without magnetization. This is allowed, because magnets without magnetization are assumed to have the same electromagnetic properties as air.
With this, the radial component of magnetization $M_r(r,\alpha_r)$

on the interval $-\pi/(2p) < \alpha_r \leq 3\pi/(2p)$ can be written as

$$M_r(r,\alpha_r) = \begin{cases} M_m \frac{r_r}{r} & \text{when } -\alpha_m < \alpha_r < \alpha_m \\ -M_m \frac{r_r}{r} & \text{when } \frac{\pi}{p} - \alpha_m < \alpha_r < \frac{\pi}{p} + \alpha_m \\ 0 & \text{on the rest of the interval } -\pi/(2p) < \alpha_r \leq 3\pi/(2p) \end{cases} \qquad (12)$$

where (see figure 2)
r_r is the radius of the rotor iron ,
α_r is the rotor coordinate,
α_m is the magnet pole arc, and
p is the number of pole pairs.
This magnetization can be written as a Fourier series, which is done because the solution of the differential equation also has the form of a Fourier series:

$$M_r(r,\alpha_r) = \sum_{k=1,3,5,..}^{\infty} \hat{M}_k \frac{r_r}{r} \cos(kp\alpha_r) \; ; \qquad (13)$$
$$\hat{M}_k = \frac{2p}{\pi} \int_{-\alpha_m}^{\alpha_m} M_m \cos(kp\alpha_r) \mathrm{d}\alpha_r = \frac{4}{k\pi} M_m \sin(kp\alpha_m)$$

B. The magnetic field due to the magnets

To calculate the magnetic vector potential due to the magnets, the differential equation (equation (8)) is solved in two regions:
- the magnet region indicated by the superscript m ($r_r < r < r_m$, see figure 2), and
- the air-gap region indicated by the superscript g ($r_m < r < r_s$, see figure 2).

Ampère's continuity condition (equation (10)) is applied to the stator surface, the rotor surface and the magnet surface. This results in three boundary conditions for the tangential component of the magnetic field strength:

$$H_{tm}^{g}(r_s,\alpha_r) = 0 \qquad (14)$$

$$H_{tm}^{m}(r_r,\alpha_r) = 0 \qquad (15)$$

$$H_{tm}^{m}(r_m,\alpha_r) = H_{tm}^{g}(r_m,\alpha_r) \qquad (16)$$

Further, the magnetic flux continuity condition (equation (9)) is applied to the magnet surface. This results in a boundary condition for the radial component of the magnetic flux density:

$$B_{rm}^{m}(r_m,\alpha_r) = B_{rm}^{g}(r_m,\alpha_r) \qquad (17)$$

With these boundary conditions and the magnetization of equation (13), the magnetic vector potential in the magnet region A_{zm}^{m} is solved as

$$A_{zm}^{m}(r,\alpha_r) = \sum_{k=1,3,5,..}^{\infty} \left(1 - \frac{(r^{kp} + r_r^{2kp} r^{-kp})(r_s^{2kp} - r_m^{2kp})}{2(r_s^{2kp} - r_r^{2kp}) r_m^{kp}} \right) \frac{r_r \mu_0 \hat{M}_k}{kp} \sin(kp\alpha_r) \qquad (18)$$

and the magnetic vector potential in the air-gap region A_{zm}^{g} is solved as

$$A_{zm}^{g}(r,\alpha_r) = \sum_{k=1,3,5,..}^{\infty} \frac{(r^{kp} + r_s^{2kp} r^{-kp})(r_m^{2kp} - r_r^{2kp})}{2(r_s^{2kp} - r_r^{2kp}) r_m^{kp}} \frac{r_r \mu_0 \hat{M}_k}{kp} \sin(kp\alpha_r) \qquad (19)$$

Figure 3 depicts the lines of constant magnetic vector potential. As shown in subsection II.C, these lines are the lines of magnetic flux. In this figure, they are caused by the magnetization of the magnets.

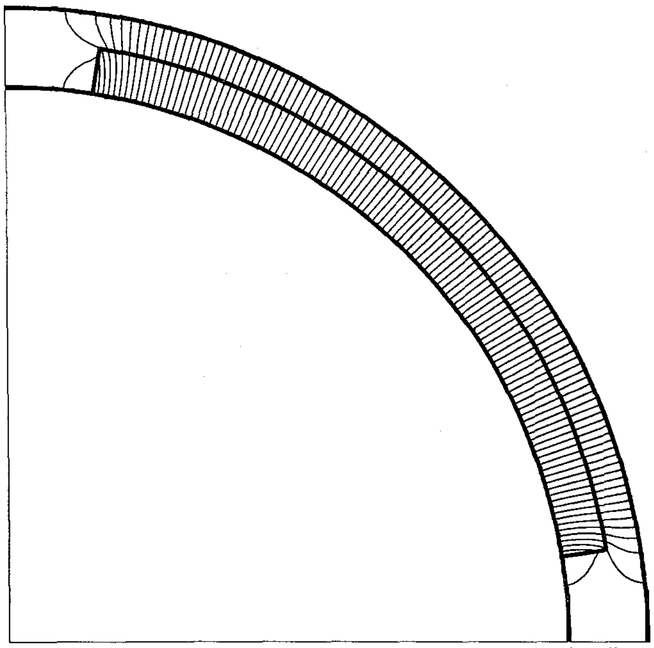

Figure 3: The lines of magnetic flux due to the magnets in the air gap and the magnets. One pole-pitch of a four-pole machine is depicted.

IV. THE STATOR CURRENTS

This section starts with a description of the surface current density due to the stator currents. This description is used for the calculation of the magnetic field due to these currents (while the magnetization of the magnets is zero).

A. The surface current density of the stator

The current in the stator slots is replaced by a surface current density K_s on the stator surface at the place of the slot openings.
The conductor density n_{sa} (the number of conductors per radian, as introduced in [5]) of stator phase a is a function of the stator coordinate α_s (see figure 2):

$$n_{sa}(\alpha_s) = \sum_{k=1,3,5,..}^{\infty} \frac{1}{2} N_{s,k} \sin(kp\alpha_s) \quad (20)$$

In this equation, $N_{s,k}$ is the number of turns of the kth space harmonic of the conductor density, which is related to the actual number of turns N by

$$N_{s,k} = \frac{4}{\pi} k_{w,k} N \sin(\tfrac{1}{2}k\pi) \quad (21)$$

where $k_{w,k}$ is the winding factor for the kth space harmonic of the actual winding.
Again, this conductor density is written as a Fourier series, because the solution of the differential equation also has the form of a Fourier series.
When a current i_{sa} flows in this winding, the surface current density of this winding can be calculated as

$$K_{sa}(\alpha_s) = \sum_{k=1,3,5,..}^{\infty} \frac{N_{s,k}}{2r_s} i_{sa} \sin(kp\alpha_s) \quad (22)$$

The conductor densities of the phases b and c are equal to the conductor density of phase a, except an angular shift of their axes, which lay at $\alpha_b = 2\pi/(3p)$ and $\alpha_c = 4\pi/(3p)$ respectively. Using this, the surface current density of a three-phase stator can be expressed as

$$K_s(\alpha_s) = \sum_{k=1,3,5,..}^{\infty} \frac{N_{s,k}}{2r_s} \cdot \left\{ i_{sa}\sin(kp\alpha_s) + i_{sb}\sin(k(p\alpha_s - \tfrac{2}{3}\pi)) + i_{sc}\sin(k(p\alpha_s - \tfrac{4}{3}\pi)) \right\} \quad (23)$$

B. The magnetic field due to the stator currents

To calculate the magnetic vector potential due to the stator currents, the differential equation (equation (8)) is solved in the air-gap region ($r_r < r < r_s$). It is not necessary to separate two regions, because magnets without magnetization are assumed to have the same electromagnetic characteristics as air.
Ampère's continuity condition (equation (10)) is applied to the stator surface and the rotor surface. This results in two boundary conditions for the tangential component of the magnetic field strength:

$$H_{ts}(r_r, \alpha_s) = 0 \quad (24)$$

$$H_{ts}(r_s, \alpha_s) = -K_s(\alpha_s) \quad (25)$$

With these boundary conditions and the surface current density of equation (23), the magnetic vector potential can be calculated as

$$A_{zs}(r, \alpha_s) = \sum_{k=1,3,5,..}^{\infty} \frac{(r^{kp} + r_r^{2kp} r^{-kp}) r_s^{kp}}{r_s^{2kp} - r_r^{2kp}} \frac{\mu_0 N_{s,k}}{2kp} \cdot \left\{ i_{sa}\sin(kp\alpha_s) + i_{sb}\sin(k(p\alpha_s - \tfrac{2}{3}\pi)) + i_{sc}\sin(k(p\alpha_s - \tfrac{4}{3}\pi)) \right\} \quad (26)$$

The lines of magnetic flux due to the stator currents are plotted in figure 4 as the lines of constant magnetic vector potential.

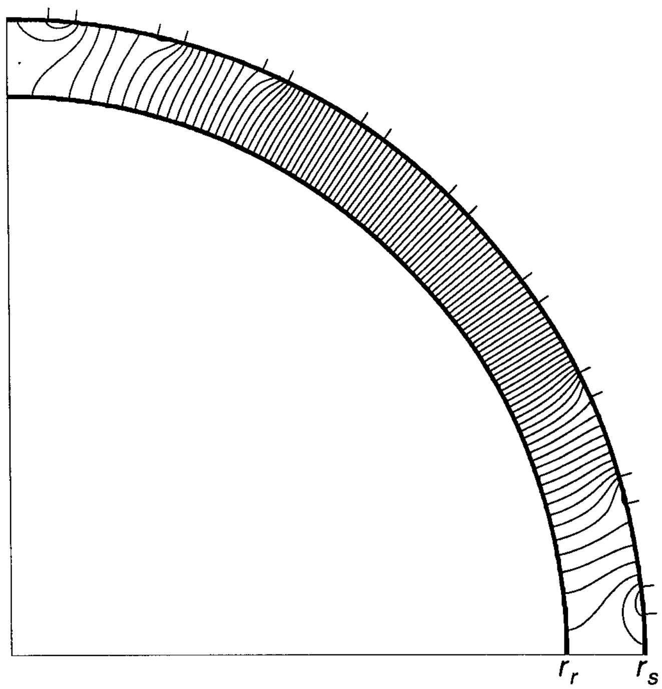

Figure 4: The lines of magnetic flux due to the stator currents ($i_{sa}=0$, $i_{sb}=-i_{sc}$) in the air gap. One pole-pitch of a four-pole machine is depicted, and the places of the slot openings are indicated.

V. THE VOLTAGE EQUATIONS

A. A general expression for the stator voltage

The general expression for the stator voltage, which follows from Faraday's law, can be written as

$$\vec{u}_s = R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} \quad (27)$$

where R_s is the stator resistance, and $\vec{u}_s$, $\vec{i}_s$, and $\vec{\psi}_s$ are vectors for the stator voltages, currents and flux linkages respectively, which are introduced as

$$\vec{u}_s = \begin{bmatrix} u_{sa} \\ u_{sb} \\ u_{sc} \end{bmatrix}; \quad \vec{i}_s = \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}; \quad \vec{\psi}_s = \begin{bmatrix} \psi_{sa} \\ \psi_{sb} \\ \psi_{sc} \end{bmatrix} \tag{28}$$

The flux linkage $\vec{\psi}_s$ of equation (27) is separated into different contributions, namely:

1) the flux linkage due to leakage fields $\vec{\psi}_{s\sigma}$, and
2) the flux linkage due to the air-gap field, which again consists of two contributions:

2a) the flux linkage due to the magnets $\vec{\psi}_{sm}$, and
2b) the flux linkage due to the stator currents $\vec{\psi}_{ss}$.

With this, the voltage equation of the stator is written as

$$\vec{u}_s = R_s\vec{i}_s + \frac{d\vec{\psi}_{s\sigma}}{dt} + \frac{d\vec{\psi}_{sm}}{dt} + \frac{d\vec{\psi}_{ss}}{dt} \tag{29}$$

In the next subsections, the flux linkages in this voltage equation are calculated. For the flux linkage due to the magnets, this is done extensively in subsection V.C. For the flux linkages due to the stator, only the results of the calculation are given.

B. The leakage flux of the stator

Because of the symmetry of the stator and the air gap, the self-inductances of the leakage flux of the stator windings are equal; they are called $L_{s\sigma a}$. For the same reason, the mutual inductances of the leakage flux between the different phases are equal; they are called $M_{s\sigma ab}$. Hence, the leakage flux can be written as

$$\vec{\psi}_{s\sigma} = \begin{bmatrix} L_{s\sigma a} & M_{s\sigma ab} & M_{s\sigma ab} \\ M_{s\sigma ab} & L_{s\sigma a} & M_{s\sigma ab} \\ M_{s\sigma ab} & M_{s\sigma ab} & L_{s\sigma a} \end{bmatrix} \vec{i}_s = \mathbf{L}_{s\sigma}\vec{i}_s \tag{30}$$

C. The flux linkages due to the magnets

The flux linkage of an arbitrary winding is related to the magnetic vector potential by using equation (5) in the general equation for the flux linkage. It is further simplified by using Stokes' integral theorem [4]:

$$\psi = \iint_S \vec{B}\cdot d\vec{a} = \iint_S \nabla\times\vec{A}\cdot d\vec{a} = \oint_C \vec{A}\cdot d\vec{s} \tag{31}$$

With this, the flux $\psi_t(\alpha_s)$ linked by a full-pitch turn at the stator surface at stator coordinate α_s can be calculated as

$$\psi_t(\alpha_s) = l_s(A_z(r_s,\alpha_s) - A_z(r_s,\alpha_s+\frac{\pi}{p})) = 2l_sA_z(r_s,\alpha_s) \tag{32}$$

where l_s is the stack length of the machine.
In this equation, we used that $A_z(r_s,\alpha_s+\pi/p) = -A_z(r_s,\alpha_s)$ is valid because of the symmetry of the machine, as appears from the derived equations for the magnetic vector potential (the equations (18), (19), and (26)).
With this expression for the flux linked by a full-pitch turn, the flux ψ_{sa} linked by stator phase a is obtained by integration:

$$\begin{aligned} \psi_{sa} &= p\int_0^{\pi/p} n_{sa}(\alpha_s)\,\psi_t(\alpha_s)\,d\alpha_s \\ &= 2p\int_0^{\pi/p} n_{sa}(\alpha_s)\,l_sA_z(r_s,\alpha_s)\,d\alpha_s \end{aligned} \tag{33}$$

The magnetic vector potential due to the magnets (equation (19)) is written as a function of the stator coordinate α_s by substituting $\alpha_r = \alpha_s - \theta$, where θ is the rotor position angle (see figure 2). When this equation with equation (20) is substituted in equation (33), the flux linkage due to the magnetic field of the magnets ψ_{sma} can be calculated as

$$\psi_{sma} = \sum_{k=1,3,5,..}^{\infty} \frac{(r_m^{2kp}-r_r^{2kp})r_s^{kp}r_r}{(r_s^{2kp}-r_r^{2kp})r_m^{kp}} \frac{\mu_0\pi\, l_sN_{s,k}\hat{M}_k}{2kp}\cos(kp\theta) \tag{34}$$

The conductor densities of the phases b and c are equal to the conductor density of phase a, except an angular shift of their axes, which lay at $\alpha_b = 2\pi/(3p)$ and $\alpha_c = 4\pi/(3p)$ respectively. Using this, the fluxes linked by the stator phases can be calculated as

$$\vec{\psi}_{sm} = \sum_{k=1,3,5,..}^{\infty} \frac{(r_m^{2kp}-r_r^{2kp})r_s^{kp}r_r}{(r_s^{2kp}-r_r^{2kp})r_m^{kp}} \frac{\mu_0\pi\, l_sN_{s,k}\hat{M}_k}{2kp} \begin{bmatrix} \cos(kp\theta) \\ \cos(k(p\theta-\frac{2}{3}\pi)) \\ \cos(k(p\theta-\frac{4}{3}\pi)) \end{bmatrix} \tag{35}$$

The magnetization of the magnets is constant. Therefore, the flux linkage due to the magnets $\vec{\psi}_{sm}$ only depends on the position of the rotor. The time derivative of this flux is the no-load voltage $\vec{e}_p$:

$$\begin{aligned} \vec{e}_p &= \frac{d\vec{\psi}_{sm}}{dt} = \sum_{k=1,3,5,..}^{\infty} -\hat{e}_{p,k} \begin{bmatrix} \sin(kp\theta) \\ \sin(k(p\theta-\frac{2}{3}\pi)) \\ \sin(k(p\theta-\frac{4}{3}\pi) \end{bmatrix} \\ \hat{e}_{p,k} &= \frac{(r_m^{2kp}-r_r^{2kp})r_s^{kp}r_r}{(r_s^{2kp}-r_r^{2kp})r_m^{kp}} \frac{\pi}{2}\mu_0 l_s\Omega N_{s,k}\hat{M}_k \end{aligned} \tag{36}$$

where Ω is the mechanical angular velocity of the rotor, which is introduced as:

$$\Omega = \frac{d\theta}{dt} \tag{37}$$

Figure 5 depicts a measured and a calculated no-load voltage, which agree good. That the horizontal parts agree, means that the important space harmonics of the magnetic field are also calculated properly. This figure affirms the usefulness of the proposed way of calculating the magnetic field.

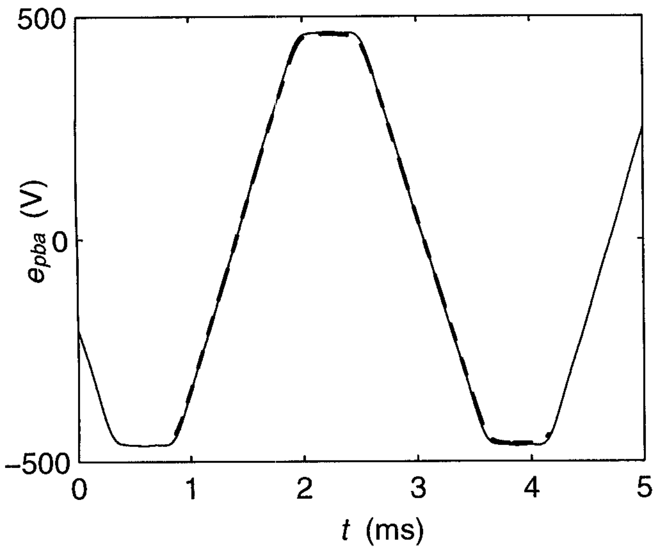

Figure 5: Measured (–) and calculated (--) no-load line voltage.

D. The fluxes linkages due to the stator currents

The fluxes linkages due to the field of the stator currents can be calculated in the same way as the flux due to the field of the magnets. This is done by using the magnetic vector potential due to the stator currents (equation (26)) instead of the magnetic vector potential due to the magnets (equation (19)). The result is

$$\vec{\Psi}_{ss} = \sum_{k=1,3,5,..}^{\infty} \mathbf{L}_{ss,k}\vec{i}_s$$

$$\mathbf{L}_{ss,k} = L_{ss,k}\begin{bmatrix} 1 & \cos(\frac{2}{3}k\pi) & \cos(\frac{2}{3}k\pi) \\ \cos(\frac{2}{3}k\pi) & 1 & \cos(\frac{2}{3}k\pi) \\ \cos(\frac{2}{3}k\pi) & \cos(\frac{2}{3}k\pi) & 1 \end{bmatrix} \quad (38)$$

$$L_{ss,k} = \frac{r_s^{2kp}+r_r^{2kp}}{r_s^{2kp}-r_r^{2kp}}\frac{\mu_0\pi l_s N_{s,k}^2}{4kp}$$

E. Summary

Using the derived expressions ((30), (36), and (38)), the voltage equation of the machine can be written as

$$\vec{u}_s = \vec{e}_p + R_s\vec{i}_s + L_{s\sigma}\frac{d\vec{i}_s}{dt} + \sum_{k=1,3,5,..}^{\infty}\mathbf{L}_{ss,k}\frac{d\vec{i}_s}{dt} \quad (39)$$

VI. CONCLUSIONS

An analytic method for the two-dimensional calculation of the magnetic field in the cylindrical air gap of a PM generator is described.

This field can be visualized by plotting the lines of constant magnetic vector potential.

The calculated magnetic field may form the basis of a machine model, as appears from the derivation of the voltage equations. A measured no-load voltage affirms the validity of the calculation method.

The proposed analytic method gives insight into the relations between dimensions and parameters of the machine, which is important when optimizing the design.

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